Nano Ideal Generalised Closed Sets in Nano Ideal Topological Spaces

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ABSTRACT
The purpose of this paper is to define and study a new class of closed sets called $NIgsemi^{*}$-closed sets in nano ideal topological spaces. Basic properties of $NIgsemi^{*}$ - closed sets are analyzed and we compared it with some existing closed sets in nano ideal topological spaces.
Key words:$NIgsemi^{*}$- closed set, closed sets in nano ideal topology ,$NIgsemi^{*}$- open set, nano topology.

1. INTRODUCTION

The concept of ideal topological space was introduced by kuratowski [9]. Also he defined the local functions in ideal topological spaces. In 1990, Jankovic and Hamlett [4] investigated further properties of ideal topological spaces. The notion of I-open sets was introduced by Jankovic et al. [5] and it was investigated by Abd El-Monsef [11]. Later, many authors introduced several open sets and generalized open sets in ideal topological spaces such as $pre\ I$-open sets[2], $semi\ I$-open sets [6], α-I-open sets[6] , αg-I-open sets [23] and gp-I-open sets [23].
In 2013, Lellis Thivagar and Carmel Richard[12] established the field of nano topological spaces which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined nano closed sets, nano-interior and nano-closure. K.Bhuvaneswari et al. [9] introduced and studied the concept of nano generalised closed sets in nano topological spaces. Later Many researchers like [3],[9] obtained several generalizations of nano open sets. In 2012, Robert et. Al [1,2] introduced the class of $semi^{*}$-open sets and $semi^{*}$-closed sets in Topological Spaces. In 2015, Paulraj Gnanachandra [19] introduced the notion of $nano\ semi^{*}$-open sets and $nano\ semi^{*}$-closed sets in terms of nano generalised closure and nano generalised interior in Nano Topological Spaces. In 2020 [18], further properties of $nano\ semi^{*}$-open sets were investigated.
M. Parimala et al. [14, 15, 17] introduced the concept of nano ideal topological spaces and investigated some of its basic properties. In 2018, M.Parimala and Jafari [15] introduced the notion of $nano\ I$-open sets and studied several properties. Further she defined nIg - open sets and nIg- closed sets in Nano Ideal Topological Spaces.
In this paper, we introduce a new type of generalized closed and open sets called $NIgsemi^{*}$-closed set and $NIgsemi^{*}$-open set in nano ideal topological spaces and investigate the relationships between this set with other sets in nano topological spaces and nano ideal topological spaces. Characterizations and properties of $NIgsemi^{*}$ -closed sets and $NIgsemi^{*}$-open sets are studied.

2. PRELIMINARIES

Throughout this paper $(U,\tau_R(X))$ (or U) represent nano topological spaces on whichno separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(U,\tau_R(X))$, $N\ cl(A)$ and $N\ int(A)$ denote the nano closure of A and the nano interior of A respectively. We recall the following definitions, which will be used in the sequel.

Definition 2.1 ([12]). Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the ***approximation space***.
Let $X \subseteq U$. Then,
(i) The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. That is, $L_R(X) = \cup \{R(x): R(x) \subseteq X, x \in U\}$ where $R(x)$ denotes the equivalence class determined by $x \in U$.
(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. That is, $U_R(X) = \cup \{R(x): R(x) \cap X \neq \varphi, x \in U\}$.
(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not -X with respect to R and is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.
Property 2.2 ([12]). If (U, R) is an approximation space and $X, Y \subseteq U$, then
(i) $L_R(X) \subseteq X \subseteq U_R(X)$
(ii) $L_R(\varphi) = U_R(\varphi) = \varphi$
(iii) $L_R(U) = U_R(U) = U$
(iv) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R Y)$
(vi) $L_R X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vii) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(viii) $L_R(X) \subseteq L_R(Y) and\ U_R(X) \subseteq U_R(Y) whenever\ X \subseteq Y$
(ix) $U(X^c) = [L_R(X)]^c\ and\ L_R(X^{\,c}) = [U_R(X)]^c$
(x) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
(xi) $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$
Definition 2.3 ([12]). Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \varphi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$ and by the property 2.2, $\tau_R(X)$ satisfies the following axioms:
(i) U and $\varphi \in \tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau_R(X)$is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$is in $\tau_R(X)$.
Therefore, $\tau_R(X)$is a topology on U called the **nano topology** on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$are called nano open sets (briefly n-open sets). The complement of a nano open set is called a ***nano closed set*** (briefly n-closed set)
Definition 2.4 [9]. Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be ***ng-closed set*** if $ncl(A) \subseteq B$ whenever $A \subseteq B \subseteq \tau_R(X)$. The complement of an ng-closed set is called a ng-open set.
Definition 2.5 [19] If $(U, \tau_R(X))$ is a Nano topological space with respect to X where$X \subseteq U$ and if $A \subseteq U$, then the nano generalized interior of $A$is defined as union of all nano g – open subsets of A and its denoted by $Nint^*(A)$. The nano generalized closure of $A$is defined as the insertion of all nano g – closed sets containing $A$and it is denoted by $\boldsymbol{Ncl^*(A)}$.
Lemma 2.6. [19] Let A be a subset of U. Then the following properties hold.
(i) $Nint(A) \subseteq Nint^*(A)$.
(ii) $Ncl^*(A) \subseteq Ncl(A)$.
(iii) $\cup Ncl^*(G_\alpha) \subseteq Ncl^*(\cup G_\alpha)$, where each G_α is nano g-closed.
Definition 2.7.[19] A subset A of a topological space $(U, \tau_R(X)$is said to be ***nano semi*-open*** [2] (briefly n-$semi^*$ open) if there is a nano open set$G$ in U such that $G \subseteq A \subseteq$

$Ncl^*(G)$ or equivalently if $A \subseteq Ncl^*(Nint(A))$. The complement of nano semi*-open set is nano semi*-closed.

Definition 2.8.[19] Let A be a subset of a nano topological space.

(i) The $nano\ semi*$-closure of A, $Ns^*cl(A) = \cap\{F: A \subseteq F, F$ is nano semi*-closed

(ii) The nano semi*-interior of $A, Ns^*int(A) = \cup\{G: G \subseteq A, G$ is nano semi*-open$\}$.

Theorem 2.9.[18] Every nano open set is $nano\ semi*$-open.

Definition 2.10[10]. An **ideal I** on a topological space (X,τ) is a nonempty collection of subsets of X, which satisfies the following two conditions:

(i) If $A \in I$ and $B \subseteq A$ implies $B \in I$

(ii) If $A \in I$ and $B \in I$, then $A \cup B \in I$

Definition 2.11[10] **An ideal topological space** is a topological space (X,τ) with an ideal I on X and it is denoted by (X,τ,I). Given a topological space (X,τ) with an ideal I on X and if $\rho(X)$ is the set of all subsets of X, a set operator $(*): \rho(X) \to \rho(X)$, called a local function of A with respect to τ and I, is defined as follows: for $A \subseteq X, A^*(I,\tau) = \{x \in X / U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau / x \in U\}$. We simply write A^* instead of $A^*(I,\tau)$.

Definition 2.12[22] For every Ideal topological space (X,τ,I), there exists a topology $\tau^*(I)$, finer than τ, generated by $\beta(I,\tau) = \{U - i / U \in \tau \,\&\, i \in I\}$. But in general (I,τ) is not always a topology. Additionally$cl^*(A) = A \cup A^*$ defines a **kuratowski closure operator** for τ* (I). If $A \subseteq X$, $cl(A)$ and $int(A)$ will, respectively, denote the closure and interior of A in (X,τ) and$int^*(A)$ denote the interior of A in (X,τ^*).

Definition 2.13 A subset A of an ideal space (X,τ,I) is ***-closed** (resp**. *-dense** in itself) if$A^* \subseteq A$ (resp. $A \subseteq A^*$).

Definition 2.14 [7] Given a space (X, τ, I) and $A \subseteq X$, A is said to be **I open** if $A \subseteq intA*$. We denoted by $IO(X,\tau) = \{A \subseteq X, A \subseteq int(A^*)\}$ or simply write $I.O$ for $IO(X,T)$ when there is no chance for confusion

Definition 2.15[17] Let $(U,\mathcal{N})$ be a nano topological space, where $\mathcal{N} = \tau_R(X)$. A nano topological spaces$(U,\mathcal{N})$ with an ideal I on $U$is called a ***nano idealtopological space*** and is denoted by $(U,\mathcal{N},I)$. Let $(U,\mathcal{N})$ be a nano topological space and $G_n(x) = \{G_n: x \in G_n, G_n \in \mathcal{N}\}$be the family of nano open sets which contain x

Definition 2.16 [17]. Let $(U,\mathcal{N},I)$ be a nano ideal topological space with an ideal I on U, where $\mathcal{N} = \tau_R(X)$ and $(.)^*_n$ be a set operator from $P(U)$ to $P(U)$. ($P(U)$ is the set of all subsets of U). For a subset $A \subset U$, $A^*_n(I,\mathcal{N}) = \{x \in U : G_n \cap A \notin I$, for every $G_n \in G_n(x)\}$, where $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$ is called ***the nano local function*** (briefly, n-local function) of A with respect to I and N. We will simply write A^*_n for $A^*_n(I,\mathcal{N})$.

Theorem 2.17 ([17]). Let $(U,\mathcal{N})$ be a nano topological space with ideals I, I' on U and A, B be subsets of U. Then

(i) $A \subseteq B \Rightarrow A^*_n \subseteq B^*_n$,

(ii) $I \subseteq I' \Rightarrow A^*_n(I') \subseteq A^*_n(I)$,

(iii) $A^*_n = n\text{-}cl(A^*_n) \subseteq n\text{-}cl(A)$ ($A^*_n$is a nano closed subset of $n\text{-}cl(A)$),

(iv) $(A^*_n)^*_n \subseteq A^*_n$,

(v) $A^*_n \cup B^*_n = (A \cup B)^*_n$

(vi) $A^*_n - B^*_n = (A - B)^*_n - B^*_n \subseteq (A - B)^*_n$,

(vii) $V \in N \Rightarrow V \cap A^*_n = V \cap (V \cap A)^*_n \subseteq (V \cap A)^*_n$ and

(viii) $J \in I \Rightarrow (A \cup J)^*_n = A^*_n = (A - J)n^*_n$.

Theorem 2.18 ([17]). If $(U,\mathcal{N},I)$ is a nano topological space with an ideal I and $A \subseteq A^*_n$, then $A^*_n = n\text{-}cl(A^*_n) = n\text{-}cl(A)$.

Definition 2.19 ([17]). Let $(U, \mathcal{N})$ be a nano topological space with an ideal I on U. The set operator $n\text{-}cl^*$ is called a **nano∗-closure** and is defined $as\ n\text{-}cl^*(A) = A \cup A_n^*$ for $A \subseteq X$. It can be easily observed that $n\text{-}cl^*(A) \subseteq n\text{-}cl(A)$.

Theorem 2.20 [17]. The set operator $n - cl^*$ satisfies the following conditions:

(i) $A \subseteq n\text{-}cl^*(A)$,
(ii) $n\text{-}cl^*(\varphi) = \varphi\ and\ n\text{-}cl^*(U) = U$,
(iii) $If\ A \subset B, then\ n\text{-}cl^*(A) \subseteq n\text{-}cl^*(B)$,
(iv) $n\text{-}cl^*(A) \cup n\text{-}cl^*(B) = n\text{-}cl^*(A \cup B)$.
(v) $n\text{-}cl^*(n\text{-}cl^*(A)) = n\text{-}cl^*(A)$.

Definition 2.21[17]. An ideal I in a space $(U, \mathcal{N}, I)$ is called**$\mathcal{N}$-codense ideal** if $\mathcal{N} \cap I = \{\varphi\}$.

Definition 2.22 [17]. A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is**n ∗-dense** in itself (resp. **n ∗-perfect and n ∗-closed**) if $A \subseteq A_n^*$ (resp. $A = A_n^*,\ A_n^* \subseteq A$).

Lemma 2.23 [17]. Let $(U, \mathcal{N}, I)$ be a nano ideal topological space and $A \subseteq U$. If A is n ∗-dense in itself, then $A_n^* = n\text{-}cl(A_n^*) = n\text{-}cl(A) = n\text{-}cl^*(A)$.

Definition 2.24.[13,15,20] A subset A of space $(U, \mathcal{N}, I)$ is said to be

1. NI open if $A \subseteq N\ int(A_n^*)$.
2. $NI\alpha$-open if $A \subseteq N\ int(Ncl^*(Nint(A)))$
3. $NI\beta$-open if $A \subseteq Ncl^*(N\ int(Ncl^*(A)))$
4. $NIsemi$-open if $A \subseteq Ncl^*(N\ int(A))$
5. $NI\ pre$-open if $A \subseteq N\ int(Ncl^*(A))$.
6. $NI\ regular$ - open if $A = N\ int\big(Ncl^*(A)\big)$

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.25[8] A subset A of space $(U, \mathcal{N}, I)$ is said to be

1. a nano ideal generalized closed set (NI_g − closed) if $NIcl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.
2. a nano ideal generalized - α closed set ($NI_{g\alpha}$ − closed) if $NI\alpha cl(A) \subseteq Z$ whenever $A \subseteq Z$ and Z is nano open.
3. a nano ideal generalized β − closed set ($NI_{g\beta}$ − closed) if $NI\beta cl(A) \subseteq Z$ whenever $A \subseteq Z$and Z is nano open.
4. a nano ideal generalized $semi$ - closed set (NI_{gs} − closed) if $NIscl(A) \subseteq Z$ whenever $A \subseteq Z$and Z is nano open
5. a nano ideal generalized pre - closed set (NI_{gp} − closed) if $NIpcl(A) \subseteq Z$ whenever $A \subseteq Z$and Z is nano open.
6. (x) a nano ideal generalized $regular$ - closed set (NI_{gr} − closed) if $NIrcl(A) \subseteq Z$ whenever $A \subseteq Z$and Z is nano open.

3. NANO IDEAL GENERALISED SEMI*-CLOSED SETS

Definition 3.1. A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is said to be ***nano ideal generalized semi*-closed*** (briefly, $NIgsemi^*$-closed) if $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is $nano\ semi^*$-open.

Definition 3.2 A subset A of a nano ideal topological space $(U, \mathcal{N}, I)$ is said to be ***nano ideal generalized semi*-open*** (briefly, $NIgsemi^*$-open) if X − A is $NIgsemi^*$-closed.

Example 3.3. Let $U = \{a, b, c, d\}$ be the universe, $X = \{a, d\} \subset U$, $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and$\mathcal{N} = \{U, \varphi, \{a, d\}\}$ and the ideal I = {φ,{a}}. Then $NIgsemi^*$-closed = $\{U, \varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}\}$

Theorem 3.4. If A and B are $NIgsemi^*$-closed , then $A \cup B$ is $NIgsemi^*$closed.

Proof. Let A and B are $NIgsemi^*$- closed sets . Then $A_n^* \subseteq V$ where $A \subseteq V$ and V is $nano\ semi^*$-open and $B_n^* \subseteq V$ where $B \subseteq V$ and V is $nano\ semi^*$- open . Since A and $B$are subsets of V, $(A_n^* \cup B_n^*) = (A \cup B)_n^*$ is a subset of V and V is n$nano\ semi^*$- open. which implies that $(A \cup B)$ is$NIgsemi^*$- closed.

Remark 3.5. The Intersection of two $NIgsemi^*$-closed sets need not be $NIgsemi^*$-closed set which is shown in the following example.

Example 3.6. Let $U = \{a,b,c,d\}$ be the universe, $X = \{c\} \subset U, U/R = \{\{a\},\{b\},\{c,d\}\}$ and $\mathcal{N} = \{U,\varphi,\{c,d\}\}$ and the ideal $I = \{\varphi,\{c\}\}$. $NIgsemi^*$ -closed = $\{U,\varphi,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,d\},\{a,c\},\{b,d\},\{a,b,c\},\{b,c,d\},\{a,b,d\},\{a,c,d\}\}$
Let $A = \{a,d\}$ and $B = \{b,d\}$ be $NIgsemi^*$-closed sets. $A \cap B = \{d\}$ is not a $NIgsemi^*$-closed set.

Theorem 3.7. If $(U,\mathcal{N},I)$ is a nano ideal topological space and $A \subseteq X$, then A is $NIgsemi^*$-closed if and only if $n\text{-}cl^*(A) \subseteq V$ whenever $A \subseteq V$ and V is$nano\ semi^*$- open in U.

Proof. Necessity: Since A is $NIgsemi^*$-closed, we have $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is $n\text{-}semi^*$open in U. Now $n\text{-}cl^*(A) = A \cup A_n^* \subseteq V$ whenever $A \subseteq V$ and V is n-open in U.
Sufficiency: Let $A \subseteq V$ and V be $nano\ semi^*$- open in U. By hypothesis $n - cl^*(A) \subseteq V$. Since $n\text{-}cl^*(A) = A \cup A_n^*$, we have $A_n^* \subseteq V$.

The following theorem gives characterizations of $NIgsemi^*$-closed sets.

Theorem 3.8. If $(U,\mathcal{N},I)$ is any nano ideal topological space and $A \subseteq U$, then the following are equivalent.
(1) A is$NIgsemi^*$-closed,
(2) $n\text{-}cl^*(A) \subseteq V$whenever $A \subseteq V$ and V is $nano\ semi^*$-open in U,
(3) For all $x \in n\text{-}cl^*(A), n\text{-}semi^*cl(\{x\}) \cap A \neq \emptyset$.
(4) $n\text{-}cl^*(A) - A$ contains no nonempty $nano\ semi^*$-closed set,
(5)$A_n^* - A$ contains no nonempty $nano\ semi^*$-closed set.

Proof. (1)⇒(2) If A is $NIgsemi^*$ -closed, then $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is $nano\ semi^*$ -open in U and so $n - cl^*(A) = A \cup A_n^* \subseteq V$ whenever $A \subseteq V$ and V is $nano\ semi^*$-open in U. This proves (2).

(2)⇒(3) Suppose $x \in n\text{-}cl^*(A)$. If $n\text{-}semi^*cl(\{x\}) \cap A = \emptyset$, then $A \subseteq U - n\text{-}semi^*cl(\{x\})$. By (2), $n\text{-}cl^*(\{x\}) \subseteq U - n\text{-}semi^*cl(\{x\})$ and hence $n\text{-}cl^*(A) \cap \{x\} = \varphi$, a contradiction, since $x \in n\text{-}cl^*(A)$.

(3)⇒(4) Suppose $F \subseteq n\text{-}cl^*(A) - A, F$ is $nano\ semi^*$-closed and $x \in F$. Since $F \subseteq U - A, F \cap A = \varphi$ .We have $n\text{-}semi^*cl(\{x\}) \cap A = \emptyset$ because F is $nano\ semi^*$-closed and $x \in F$. From (3), this is a contradiction. Therefore $n\text{-}cl^*(A) - A$ contains no nonempty $n\text{-}semi^*$closed set.

(4)⇒(5) Since $n\text{-}cl^*(A) - A = (A \cup A_n^*) - A = (A \cup A_n^*) \cap A^C = (A \cap A^c) \cup (A_n^* \cap A^C) = A_n^* \cap A^C = A_n^* - A$. Therefore $A_n^* - A$ contains no nonempty $nano\ semi^*$- closed set.

(5)⇒(1) Let $A \subseteq V$ where V is $nano\ semi^*$- open set. Therefore U$-V \subseteq U - A$ and so $A_n^* \cap (U - V) \subseteq A_n^* \cap (U - A) = A_n^* \cap A^C = A_n^* - A$. Therefore $A_n^* \cap (U - V) \subseteq A_n^* - A$. Since A_n^* is always n-closed set, so $A_n^*$is $nano\ semi^*$-closed set and so $A_n^* \cap (U - V)$ is a $nano\ semi^*$-closed set contained in $A_n^* - A$. Therefore $A_n^* \cap (U - V) = \emptyset$ and hence$A_n^* \subseteq V$. Therefore$A$ is $NIg\ semi^*$-closed

Corollary 3.9. Let $(U,\mathcal{N},I)$ be a nano ideal topological space and $A \subseteq U$ is an $NIg\ semi^*$-closed set, then the following are equivalent:
(i) A is an n^*-closed set.
(ii) $n\text{-}cl^*(A) - A$ is an $n\text{-}semi^*$closed set.

(iii) $A_n^* - A$ is an n-$semi$ *closed set.

Proof. (i) ⇒ (ii) If A is n^*-closed, then $A_n^* \subseteq A$ and so n-$cl^*(A) - A = \varphi$. Hence n-$cl^*(A) - A$ is n-$semi$ *closed.

(ii) ⇒ (iii) Since n-$cl^*(A) - A = A_n^* - A$, it is clear.

(iii) ⇒ (i) If $A_n^* - A$ is n-$semi$ *closed and A is n-$semi$ *closed, from Theorem 3.5(v), $A_n^* - A = \varphi$ and so A is n^*-closed.

Theorem 3.10. If A is NIg $semi$ *-closed and A$\subseteq B \subseteq A_n^*$, then B is n-$semi$ *closed.

Proof. Let $B \subseteq V$ where V is n-$semi$ *open in $\mathcal{N}$. Then $A \subseteq B$ implies $A \subseteq V$. Since A is NIg $semi$ *-closed, $A_n^* \subseteq V$. Also $B \subseteq A_n^*$ implies $B_n^* \subseteq A_n^*$. Thus $B_n^* \subseteq V$ and so B is NIg $semi$ *-closed.

Theorem 3.11. Let $(U, \mathcal{N}, I)$) be a nano ideal space. Then every subset of U is NIg $semi$ *-closed if and only if every n-$semi$ *open set is n^* -closed.

Proof: Suppose every subset of U is NIg $semi$ * -closed. If $G \subseteq U$, U is n-$semi$ *open then G is NIg $semi$ *-closed and so $(G)_n^* \subseteq G$. Hence G is n^* -closed. Conversely, suppose that every n-$semi$ *open set is n^*-closed. If G is n-$semi$ *open set such that $A \subseteq G \subseteq U$ then $(A)_n^* \subseteq (G)_n^* \subseteq G$ and so A is NIg $semi$ *-closed.

Theorem 3.12. If$(U, \mathcal{N}, I)$ is any nano ideal topological space where $I = \{\varphi\}$, then A is NIg $semi$ *-closed if and only if A is ng -closed.

Proof. The proof follows from the fact that for $I = \{\varphi\}, A_n^* = n$-$cl(A) \supset A$ and so every subset of U is n^*-dense in itself.

Theorem 3.13. Let $(U, \mathcal{N}, I)$ be a nano ideal topological space. Then every subset of U is NIg $semi$ *-closed if and only if every n-$semi$ *open set is n^*-closed.

Proof. Suppose every subset of U is NIg $semi$ *-closed. If V is n-$semi$ *open, then V is NIg $semi$ *-closed and so $V_n^* \subseteq V$. Hence V is n^*-closed. Conversely, suppose that every n-$semi$ *open set is n^*-closed. If $A \subseteq U$ and V is an n-$semi$ *open set such that $A \subseteq V$, then $A_n^* \subseteq V_n^*$ and so A is NIg $semi$ *-closed.

Theorem 3.14. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space. For every $A \in I$, A is NIg $semi$ *-closed.

Proof. Let $A \subseteq U$ where U is n-$semi$ *open set. Since $A_n^* = \emptyset$ for every $A \in I$, then n-$cl^*(A) = A \cup A_n^* = A \subseteq U$. Therefore, by Theorem 3.7, A is NIg $semi$ *-closed.

Theorem 3.15. If $(U, \mathcal{N}, I)$ is an nano ideal topological space, then A_n^* is always NIg $semi$ *-closed for every subset A of U.

Proof. Let $A_n^* \subseteq U$ where U is n-$semi$ *open. *Since* $(A_n^*)_n^* \subseteq A_n^*$, we have $(A_n^*)_n^* \subseteq U$ whenever $A_n^* \subseteq U$ and U is n-$semi$ *open. Hence A_n^* is NIg $semi$ *-closed

Theorem 3.16. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space and $A \subseteq K$. Then A is NIg $semi$ *-closed if and only if $A = F - M$ where F is n^*-closed and M contains no nonempty n-$semi$ *closed set.

Proof. If A is NIg $semi$ *-closed, then by Theorem 3.8 (5), $M = A_n^* - A$ contains no nonempty n-$semi$ *closed set. If $F = ncl^*(A)$, then F is n^*- closed such that $F - M = (A \cup A_n^*) - (A_n^* - A) = (A \cup A_n^*) \cap (A_n^* \cap A^c)^c = (A \cup A_n^*) \cap ((A_n^*)^c \cup A) = (A \cup A_n^*) \cap (A \cup (A_n^*)^c) = A \cup ((A_n^* \cap A^c)^c \cap (A_n^*)^c) = A$.

Conversely, suppose $A = F - M$ where F is n^*- closed and M contains no nonempty n-$semi$ *closed set. Let V be an n-$semi$ *open set such that $A \subseteq V$. Then $F - M \subseteq V$ which implies that $F \cap (U - V) \subseteq M$. Now $A \subseteq F$ and $F_n^* \subseteq F$ then $A_n^* \subseteq F_n^*$ and so $A_n^* \cap (U - V) \subseteq F_n^* \cap (U - V) \subseteq F \cap (U - V) \subseteq M$. By hypothesis, since $A_n^* \cap (U - V)$ is n-$semi$ *closed, $A_n^* \cap (U - V) = \emptyset$ and so $A_n^* \subseteq V$. Hence A is NIg $semi$ *-closed.

Theorem 3.17. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space and $A \subseteq U$. If $A \subseteq B \subseteq A_n^*$, then $A_n^* = B_n^*$ and B is n^*-dense in itself.

Proof. Since $A \subseteq B$, then $A_n^* \subseteq B_n^*$ and since $B \subseteq A_n^*$, then $B_n^* \subseteq (A_n^*)_n^* \subseteq A_n^*$ by Theorem 2.17 (4). Therefore $A_n^* = B_n^*$ and $B \subseteq A_n^* \subseteq B_n^*$. Hence proved.

Theorem 3.18. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space. If A and B are subsets of U such that $A \subseteq B \subseteq n\text{-}cl^*(A)$ and A is $NIg\ semi^*$-closed, then B is $NIg\ semi^*$-closed.

Proof. Since A is $NIg\ semi^*$-closed, then by Theorem 3.8 (4), $n\text{-}cl^*(A) - A$ contains no nonempty $n\text{-}semi^*$ closed set. Since $n\text{-}cl^*(B) - B \subseteq n\text{-}cl^*(A) - A$, $n\text{-}cl^*(B) - B$ contains no nonempty $n\text{-}semi^*$closed set and so by Theorem 3.8 (4), B is $NIg\ semi^*$-closed

Theorem 3.19. If $(U, \mathcal{N}, I)$ is any nano ideal topological space, then everynano ideal generalised closed is $NIg\ semi^*$-closed set but not conversely.

Proof. Let $(U, \mathcal{N}, I)$ be an nano ideal topological space.Let $A \subseteq U$ and U is $nano\ semi^*$-open. If A is nano ideal generalised closed set then $A_n^* \subseteq V$ whenever $A \subseteq V$ and V is $nano$ open set. We know the fact that every nano open set is $n\text{-}semi^*$ open. Thereforeb $A_n^* \subseteq V \subseteq U$. Hence everynano ideal generalised closed is $NIg\ semi^*$-closed set

Theorem 3.20. If $(U, \mathcal{N}, I)$ is any nano ideal topological space, then every ng -closed set is $NIg\ semi^*$-closed but not conversely

Proof. Let V be any nano open set containing A.Since every nano open set is $n\text{-}semi^*$open, V be any $n\text{-}semi^*$ open set containing$A$. ASince A is ng -closed, $n\text{-}cl(A) \subseteq V$. By Theorem 2.17(iii), we have $A_n^* \subseteq V$. Then A is $NIg\ semi^*$- closed set.

Example 3.21. Let U = {a,b,c,d}, with U/ R= {{b},{d},{a,c}} and X= {a,d},. Then the Nano topology $\mathcal{N} = \{\phi, U, \{d\}, \{a, c\}, \{a.c.d\}\}$ and $I = \{\phi, \{d\}\}$.Then $NIg\ semi^*$-closed sets are {ϕ, U, {b}, {d},{a,b},{b,c},{b,d},{a,b,c},{b,c,d},{a,b,d}} and ng -closed sets are {ϕ, U, {b}, {a,b},{b,c},{b,d},{a,b,c},{b,c,d},{a,b,d}. Since {d} is $NIg\ semi^*$-closed but it is not ng -closed,every nano generalised closed set is $NIg\ semi^*$-closed

Theorem 3.22. Let $(U, \mathcal{N}, I)$ be a nano ideal topological space. Every n∗ -closed set is $NIg\ semi^*$-closed.

Proof. Let A be a subset of X and A be n∗ -closed. Assume that A ⊆ V and V is $n\text{-}semi^*$open. Since A is n∗ -closed, we have $A_n^* \subseteq A$ and so A is $NIg\ semi^*$-closed.

Example 3.23. Let U = {a,b,c,d}, with U/ R= {{a},{b},{c,d} and X= {c},. Then the Nano topology $\mathcal{N} = \{\phi, U, \{c, d\}\}$ and $I = \{\phi, \{c\}\}$ .Then $NIg\ semi^*$ -closed sets = {ϕ, U,{a},{b},{c},{a,b},{b,c},{a,c},{a,d},{b.d},{a,b,c},{b,c,d},{a,b,d},{a,c,d}} and n^*- closed sets are {ϕ, U,{a,b},{a,b,c}}. It is clear that every n∗ -closed set is $NIg\ semi^*$-closed.

For the relationship related to several sets defined in this paper, we have the following diagram:

n-closed ng - closed ⟹ $NIg\ semi^*$-closed nIg - closed ⟸

↑

n^*-dense in itself n^*perfect n^*-closed⟹

The following theorem gives a characterization of $NIg\ semi^*$-open sets.

Theorem 2.31. Let be an nano ideal topological space and $A \subseteq K$. Then A is $NIg\ semi^*$-open if a$(U, \mathcal{N}, I)$nd only if $F \subseteq n\text{-}int^*(A)$ whenever F is $n\text{-}semi^*$closed and $F \subseteq A$.

Proof. Suppose A is $\mathcal{N}\mathcal{I}g$ $semi^*$-open. If F is n-$semi^*$closed and $F \subseteq A$, then $U - A \subseteq U - F$ and so n-$cl^*(U - A) \subseteq U - F$ by Theorem 2.4 (2). Therefore $F \subseteq U - n$-$cl^*(U - A) = n$-$int^*(A)$. Hence $F \subseteq n$-$int^*(A)$.
Conversely, suppose the condition holds. Let V be a n-$semi^*$open set such that $U - A \subseteq V$. Then $U - V \subseteq A$ and so $U - V \subseteq n$-$int^*(A)$. Therefore n-$cl^*(U - A) \subseteq V$. By Theorem 2.4 (2), $U - A$ is $\mathcal{N}\mathcal{I}g$ $semi^*$-closed. Hence A is $\mathcal{N}\mathcal{I}g$ $semi^*$-open.
Theorem 3.18. Let $(U, \mathcal{N}, \mathcal{I})$ be a nano ideal topological space and $A \subseteq U$. Then A is $\mathcal{N}\mathcal{I}g$ $semi^*$-open if and only if $F \subseteq n$-$int^*(A)$ whenever F is closed and $F \subseteq A$.
Proof. Suppose A is $\mathcal{N}\mathcal{I}g$ $semi^*$-open. If F is closed and $F \subseteq A$, then $U - A \subseteq U - F$ and so n-$cl^*(U - A) \subseteq U - F$. Therefore, $F \subseteq n$-$int^*(A)$.
Conversely, suppose the condition holds. Let V be an open set such that $U - A \subseteq V$. Then $U - V \subseteq A$ and so $U - V \subseteq n$-$int^*(A)$ which implies that n-$cl^*(U - A) \subseteq V$. Therefore, $U - A$ is $\mathcal{N}\mathcal{I}g$ $semi^*$-closed and so A is $\mathcal{N}\mathcal{I}g$ $semi^*$-open.
The following theorem gives a property of $\mathcal{N}\mathcal{I}g$ $semi^*$-closed.
Theorem 2.33. Let $(U, \mathcal{N}, \mathcal{I})$ be an nano ideal topological space and $A \subseteq U$. If A is $\mathcal{N}\mathcal{I}g$ $semi^*$-open and n-$int^*(A) \subseteq B \subseteq A$, then B is $\mathcal{N}\mathcal{I}g$ $semi^*$-open.
Proof. Since A is $\mathcal{N}\mathcal{I}g$ $semi^*$-open, then U−A is $\mathcal{N}\mathcal{I}g$ $semi^*$-closed. By Theorem 2.4 (4), n-$cl^*(U - A) - (U - A)$ contains no nonempty n-$semi^*$closed set. Since n-$int^*(A) \subseteq n$-$int^*(B)$ which implies that n-$cl^*(U - B) \subseteq n$-$cl^*(U - A)$ and so n-$cl * (U - B) - (U - B) \subseteq n$-$cl^*(U - A) - (U - A)$. Hence B is $\mathcal{N}\mathcal{I}g$ $semi^*$-open.
The following theorem gives a characterization of $\mathcal{N}\mathcal{I}g$ $semi^*$-closed sets in terms of $\mathcal{N}\mathcal{I}g$ $semi^*$-open sets.
Theorem 2.34. Let $(U, \mathcal{N}, \mathcal{I})$ be an nano ideal topological space and $A \subseteq U$. Then the following are equivalent.
(1) A is $\mathcal{N}\mathcal{I}g$ $semi^*$-open
(2) $A \cup (U - A_n^*)$ is $\mathcal{N}\mathcal{I}g$ $semi^*$-closed,
(3) $A_n^* - A$ is $\mathcal{N}\mathcal{I}g$ $semi^*$-open.
Proof. (1)⇒(2) Suppose A is $\mathcal{N}\mathcal{I}g$ $semi^*$-open. If V is any n-$semi^*$open set such that $A \cup (U - A_n^*) \subseteq V$, then $U - V \subseteq U - (A \cup (U - A_n^*)) = U \cap (A \cup (A_n^*)^c)^c = A_n^* \cap A^c = A_n^* - A$. Since A is $\mathcal{N}\mathcal{I}g$ $semi^*$-closed, by Theorem 2.4 (5), it follows that $U - V = \emptyset$ and so $U = V$. Therefore $A \cup (U - A_n^*) \subseteq V$ which implies that $A \cup (U - A_n^*) \subseteq U$ and so $(A \cup (U - A_n^*)_n^* \subseteq U_n^* \subseteq U = V$. Hence $A \cup (U - A_n^*)$ is $\mathcal{N}\mathcal{I}g$ $semi^*$-closed.
(2)⇒(1) Suppose $A \cup (U - A_n^*)$ is $\mathcal{N}\mathcal{I}g$ $semi^*$-closed. If F is any n-$semi^*$closed set such that $F \subseteq A_n^* - A$, then $F \subseteq A_n^*$ and F is not a subset of A which implies that $U - A_n^* \subseteq U - F$ and $A \subseteq U - F$. Therefore $A \cup (U - A_n^*) \subseteq A \cup (U - F) = U - F$ and $U - F$ is n-$semi^*$open. Since $(A \cup (U - A_n^*))_n^* \subseteq K - F$ which implies that $A_n^* \cup (U - A_n^*))_n^* \subseteq U - F$ and so $A_n^* \subseteq U - F$ which implies that $F \subseteq U - A_n^*$. Since $F \subseteq A_n^*$, it follows that $F = \emptyset$. Hence A is $\mathcal{N}\mathcal{I}g$ $semi^*$-open.
(2)⇔(3) Since $U - (A_n^* - A) = U \cap (A_n^* \cap A^c)^c = U \cap ((A_n^*)^c \cup A) = (U \cap (A_n^*)^c) \cup (U \cap A) = A \cup (U - A_n^*)$ is $\mathcal{N}\mathcal{I}g$ $semi^*$-closed. Hence $A_n^* - A$ is $\mathcal{N}\mathcal{I}g$ $semi^*$-open

CONCLUSIONWe defined the concept of $\mathcal{N}\mathcal{I}g$ $semi^*$-open sets and $\mathcal{N}\mathcal{I}g$ $semi^*$-closed sets in nano ideal topological spaces. We also discussed some of their properties with suitable examples. We hope that this paper is just a beginning of a new structure. It will inspire many to contribute to the cultivation of nano ideal topology in the field of Mathematics.

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