Pythagorean Fuzzy HX Bi – Ideals in HX- Near Rings

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Abstract

This article introduces the notion of Pythagorean Fuzzy HX Bi-Ideals in HX Near Rings and delves into their relevant properties. Moreover, we examine the homomorphic image and pre-image of these bi-ideals and analyze various related theorems. Throughout our analysis, we find that Pythagorean fuzzy HX bi-ideals play a crucial role in the study of HX Near Rings, and offer valuable insights into their structure and properties. By exploring the various properties and theorems associated with these Bi-Ideals, we gain a deeper understanding of their significance and potential applications in a wide range of fields. Overall, our research contributes to the ongoing exploration and development of HX Near Rings, and opens up exciting new avenues for future research in this area. We hope that our findings will inspire further study and investigation, and contribute to the advancement of this important field of mathematics.

Key words*:* HX Near Ring, HX Bi – Ideal, Pythagorean Fuzzy Set, Pythagorean Fuzzy HX Bi – Ideal,Supremum Property.

A. Introduction:

In his seminal paper [6], L.A. Zadeh introduced fuzzy set theory and its relationship to traditional set theory, providing a framework for generalizing fundamental algebraic concepts. Building on this foundation, subsequent researchers have developed a range of related concepts and structures, including Pythagorean Fuzzy Subsets [2], Intuitionistic Q Fuzzy Bi-Ideals in Near-Rings [1], Pythagorean Fuzzy Ideal semigroups [5], anti-fuzzy HX Bi-Ideals of HX Rings [3], and Intuitionistic Fuzzy HX Bi-Ideals of HX Rings [4], Intuitionistic Anti - Fuzzy HX Bi-Ideal of a HX Ring [7].This article aims to add to the ongoing research in algebraic structures by presenting a novel idea - the Pythagorean Fuzzy HX Bi-Ideal of a HX Near Ring. Our analysis includes an investigation of various related properties and characteristics of this structure, providing a deeper understanding of its potential applications and significance in the field. Additionally, we explore the image and pre-image of Pythagorean Fuzzy HX Bi-Ideals, examining their properties and connections to other algebraic concepts. Through our analysis, we find that Pythagorean Fuzzy HX Bi-Ideals offer valuable insights into the structure and properties of HX Near Rings, and provide a powerful framework for exploring and generalizing a range of fundamental algebraic concepts. Overall, our research contributes to the ongoing exploration and development of algebraic structures, offering new insights and potential applications in a range of fields. We hope that our findings will inspire further study and investigation, and contribute to the ongoing advancement of this important area of mathematics.

B. Preliminaries

DEFINITION B1: [4]

Consider a ring R and a non-empty subset $\vartheta \subset 2R - \{\varphi\}$ equipped with binary operations '+' and '.'. If ϑ forms a ring under the algebraic operations defined by:

(i) a + b = $\{a_1 + b_1 / a_1 \in$ a and $b_1 \in$ b}, where Q represents the null element, and –a represents the negative element of a.

(ii) ab = { $a_1 b_1 / a_1 \in$ a and $b_1 \in$ b},

(iii) a (b + c) = ab+ ac and (b + c) a = ba + ca

DEFINITION B2: [4]

Consider a ring R and an Intuitionistic Fuzzy Set H on R, where H is given by the expression H = ⟨x, μ(x), η(x)⟩ / x∈R}, where μ : R → [0,1] and η : R → [0,1], and 0 ≤ μ(x) + η(x) ≤ 1 for all x in R. Let ℜ be a HX ring, a non-empty subset of 2R − {φ} with binary operations '+' and '.' that satisfy the properties mentioned in the previous article. An intuitionistic fuzzy subset λH = {⟨a, λμ(a), λη(a)⟩ / a∈ℜ and 0 ≤ λμ(a) + λη(a) ≤ 1} of ℜ is called an Intuitionistic Fuzzy HX Bi-Ideal or Intuitionistic Fuzzy Bi-Ideal induced by H of the HX ring ℜ if the following conditions are met for all a, b, and c in ℜ:

(i) λμ(a−b) ≥ min {λμ(a), λμ(b)}

(ii)λμ(ab) ≥ min {λμ(a), λμ(b)}

(iii) λμ(abc) ≥ min {λμ(a), λμ(c)}

(iv) λη(a−b) ≤ max {λη(a), λη(b)}

(v) λη(ab) ≤ max {λη(a), λη(b)}

(vi) λη(abc) ≤ max {λη(a), λη(c)}

where λμ (a) = max{ μ(x) / for all x∈ a⊆ R} and λη (a) = min{η(x) / for all x∈a ⊆ R}.

DEFINITION B3: [5]

Consider a universe of discourse X and a Pythagorean Fuzzy Set (PFS) P defined as P = {z, ϑ(x), ωp(x) / z ∈ X}, where ϑ: X → [0,1] and ω: X → [0,1] represent the degree of membership and non-membership of the object z ∈ X to the set P, subject to the condition 0 ≤ (ϑp(z)) + (ωp(z))² ≤ 1 for all z in X. For simplicity, a PFS can be denoted as P = (ϑ(z), ωp(z)).

C.MAIN RESULTS:

DEFINITION: C1

Let N be a Near Ring, and let $\vartheta \subset 2^N - \{\emptyset\}$ be a non-empty subset of $2^N - \{\emptyset\}$. We say that ϑ is an HX-Near Ring on N if it satisfies the following conditions:

(i)$(N,+)$ is a group (may or may not be Abelian)

(ii) $(N, \cdot)$ be a semi group

(iii)W+I ={w +i/ w ∈ W and i ∈ I} assuming that 'w' is a variable representing an element in the ring R: Let Q denote the null element of R, and let -w denote the negative element of the element w in R.

(iv)WI={wi/ w∈ W and i ∈ I}

(v)w(i + k)= wi+ wk and(i + k)w = iw+ik

DEFINITION: C2

Let X be a universe of discourse, A Pythagorean Fuzzy set P={k, $\varrho_p(k), \zeta_p(k) / k \in$ w}. Where , $\varrho_p: X \longrightarrow$ [0,1] and $\zeta_p: X \longrightarrow$ [0,1] represent the membership and non - membership of the object k∈ w to the subset P such that 0 ≤ w is denoted as $P = (\varrho_p(\mathrm{w}), \zeta_p(\mathrm{w}))$.

DEFINITION: C3

Let N be a HX – Near Ring. Let E = $\{(w, \varrho(w), \zeta(w)/w \in N)\}$ be a Pythagorean Fuzzy Set defined on a HX- Near Ring N, where $\varrho_p: X \longrightarrow [0,1]$ and $\zeta_p: X \longrightarrow [0,1]$ such that $0 \leq \left(\varrho_p(w)\right)^2 + \left(\zeta_p(w)\right)^2 \leq 1$ let $\vartheta \subset 2^N - \{\emptyset\}$ be a HX- Near Ring. An Pythagorean Fuzzy Subset $\gamma_{P_E} = \left\{\left(w, \varrho_{P_E}(w), \zeta_{P_E}(w)\right)\right)/w \in N\}$andc0 $\leq\left(\varrho_{P_E}(w)\right)^2 + \left(\zeta_{P_E}(w)\right)^2 \leq 1$ of N is said that a Pythagorean Fuzzy HX-Bi – Ideals or Pythagorean Fuzzy Bi – Ideals induced by E of a HX- Near Ring N if Suppose the following conditions hold true: $\forall\ w, i, k \in N$

(PYI) $\varrho_{P_E}(w - i) \succcurlyeq min\{\varrho_{P_E}((w), \varrho_{P_E}((i)\}$

(PYII) $\varrho_{P_E}((wik) \succcurlyeq min\{\varrho_{P_E}(w), \varrho_{P_E}((i), \varrho_{P_E}(k)\}$

(PYIII) $\zeta_{P_E}(w - i) \preccurlyeq max\{\zeta_{P_E}(w), \zeta_{P_E}(i)\}$

(PYIV) $\zeta_{P_E}(wik) \preccurlyeq max\{\zeta_{P_E}(w), \zeta_{P_E}(i), \zeta_{P_E}(k)\}$

where $\varrho_{P_E}(w) = \sup\{\varrho_p(w)/\forall w \in A \subseteq N\}$ and $\zeta_{P_E}(w) = inf\{\zeta_p(w)/\forall w \in A \subseteq N\}$ for $'\succcurlyeq'$ means that $'\geq'$ and $'\preccurlyeq'$ means that $'\leq'$

EXAMPLE: C4

Let $N = \{a, b, c, d\}$ be set with two binary operations as follows: Then $(N, +, \cdot)$ is HX- Near Ring. We define

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

·	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	b
d	a	a	a	b

$\varrho_{P_E}(a) = sup\{\varrho_p(w)/w \in a \subseteq N\} = 0.8$, $\varrho_{P_E}(b) = sup\{\varrho_p(w)/w \in b \subseteq N\} = 0.7$

$\varrho_{P_E}(c) = sup\{\varrho_p(w)/w \in c \subseteq N\} = 0.5$, $\varrho_{P_E}(d) = sup\{\varrho_p(w)/w \in d \subseteq N\} = 0.5$

$\zeta_{P_E}(a) = inf\{\zeta_p(w)/w \in a \subseteq N\} = 0.1$, $\zeta_{P_E}(b) = inf\{\zeta_p(w)/w \in b \subseteq N\} = 0.2$

$\zeta_{P_E}(c) = inf\{\zeta_p(w)/w \in c \subseteq N\} = 0.4$, $\zeta_{P_E}(d) = in\{\zeta_p(w)/w \in d \subseteq N\} = 0.4$

Clearly N is a HX Bi – Ideal of a HX Near Ring.

THEOREM: C5

Suppose ɢ and ʜ are Pythagorean Fuzzy Sets on a HX Near Ring N. Let $\varrho_{P_ɢ}$ and $\zeta_{P_ʜ}$ be Pythagorean Fuzzy HX-Bi-Ideals in N. Then, their union, denoted by $\varrho_{P_ɢ} \cup \zeta_{P_ʜ}$ is also a Pythagorean Fuzzy HX-Bi-Ideal in N.

PROOF:

Suppose ɢ $=\{< w, \eta_p(w), \gamma_p(w) >/w \in N\}$ and ʜ$=\{< w, \varrho_p(w), \zeta_p(w) >/w \in N\}$ be any two Pythagorean Fuzzy Sets defined on a HX Near Ring N . Then $\varrho_{P_ɢ} = \{<$ w, ϱ_{P_γ} (w), ϱ_{P_δ} (w) / w $\in N\}$ and $\zeta_{P_ʜ} = \{<$ w, ζ_{P_α} (w),ζ_{P_β} (w) / w $\in N\}$ be any two Pythagorean Fuzzy HX Bi – Ideal of a HXNear Ring N. Then $\varrho_{P_ɢ} \cup \zeta_{P_ʜ}$ = {w, $(\varrho_{P_\gamma}\ \zeta_{P_\alpha})$(w), $(\varrho_{P_\delta} \cap \zeta_{P_\beta})$(w)/w $\in N\}$

Let

(i) $(\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(w - i) = max\{\varrho_{P_\gamma}(w - i), \zeta_{P_\alpha}(w - i)\}$

$\succcurlyeq max\{min\{\varrho_{P_\gamma}(w), \varrho_{P_\gamma}(i)\}, min\{\zeta_{P_\alpha}(w), \zeta_{P_\alpha}(i)\}\}$

$= max\{min\{\varrho_{P_\gamma}(w), \zeta_{P_\alpha}(w)\}, min\{\varrho_{P_\gamma}(i), \zeta_{P_\alpha}(i)\}\}$

$=\min\{max\{\{\varrho_{P_\gamma}(w), \zeta_{P_\alpha}(w)\}, max\{\varrho_{P_\gamma}(i), \zeta_{P_\alpha}(i)\}\}\}$

Therefore

$(\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(w-i) \geqslant min\{(\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(w), (\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(i)\}$

(ii) $(\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(wik) = max\{\varrho_{P_\gamma}(wik), \zeta_{P_\alpha}(wik)\}$

$\geqslant max\{\min\{\varrho_{P_\gamma}(w), \varrho_{P_\gamma}(i), \varrho_{P_\gamma}(k)\}, min\{\zeta_{P_\alpha}(w), \lambda_p^\alpha(i), \zeta_{P_\alpha}(k)\}\}$

$=max\{\min\{\varrho_{P_\gamma}(w), \zeta_{P_\alpha}(w)\}, min\{\varrho_{P_\gamma}(i), \zeta_{P_\alpha}(i)\}, min\{\varrho_{P_\gamma}(k), \zeta_{P_\alpha}(k)\}\}$

$= \min\{max\{\{\varrho_{P_\gamma}(w), \zeta_{P_\alpha}(w)\}, max\{\varrho_{P_\gamma}(i), \zeta_{P_\alpha}(i)\}, max\{\varrho_{P_\gamma}(k), \zeta_{P_\alpha}(k)\}\}\}$Therefore

$(\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(wik) \geqslant min\{(\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(w), (\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(i), (\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(k)\}$

(iii) $(\varrho_{P_\delta} \cap \zeta_{P_\beta})(w-i) = min\{\varrho_{P_\delta}(w-i), \zeta_{P_\beta}(w-i)\}$

$\leqslant min\{max\{\varrho_{P_\delta}(w), \varrho_{P_\delta}(i)\}, max\{\zeta_{P_\beta}(w), \zeta_{P_\beta}(i)\}\}$

$=min\{\max\{\varrho_{P_\delta}(w), \zeta_{P_\beta}(w)\}, \max\{\varrho_{P_\delta}(i), \zeta_{P_\beta}(i)\}\}$ $\leqslant min\{\max\{\varrho_{P_\delta}$ $(w), \zeta_{P_\beta}(w)\}, \max\{\varrho_{P_\delta}(i), \zeta_{P_\beta}(i)\}$

$\leqslant max\{\min\{\varrho_{P_\delta}(w), \zeta_{P_\beta}(w)\}, \min\{\varrho_{P_\delta}(i), \zeta_{P_\beta}(i)\}\}$

Therefore

$(\varrho_{P_\delta} \cap \zeta_{P_\beta})(w-i) \leqslant \max\{((\varrho_{P_\delta} \cap \zeta_{P_\beta}))(w), (\varrho_{P_\delta} \cap \zeta_{P_\beta}))(i)\}$

(iv)$(\varrho_{P_\delta} \cap \zeta_{P_\beta})(wik) = min\{\varrho_{P_\delta}(wik), \zeta_{P_\beta}(wik)\}$

$\leqslant min\{\max\{\varrho_{P_\delta}(w), \varrho_{P_\delta}(i), \varrho_{P_\delta}(\mathrm{k})\}, max\{\zeta_{P_\beta}(w), \zeta_{P_\beta}(i), \zeta_{P_\beta}(\mathrm{k})\}\}$

$=max\{\min\{\varrho_{P_\delta}(w), \zeta_{P_\beta}(w)\}, min\{\varrho_{P_\delta}(i), \zeta_{P_\beta}(i)\}, min\{\varrho_{P_\delta}(k), \zeta_{P_\beta}(k)\}\}$

Therefore

$(\varrho_{P_\delta} \cap \zeta_{P_\beta})(wik) \leqslant \max\{(\varrho_{P_\delta} \cap \zeta_{P_\beta})(w), (\varrho_{P_\delta} \cap \zeta_{P_\beta})(i), (\varrho_{P_\delta} \cap \zeta_{P_\beta})(k)\}$

Hence

$\varrho_{P_G} \cup \zeta_{P_H}$ is a Pythagorean fuzzy HX- bi – ideal of a HX Near ring N.

THEOREM: C6

Let N_1 and N_2 be any two HX Near Rings on the HX Near Ring N_1 and N_2 respectively. Let $f: N_1 \to N_2$ be an homomorphism onto HX Near Rings. Let H be a Pythagorean Fuzzy Subset of N_1. Let ζ_{P_H}be a Pythagorean Fuzzy HX Bi – Ideal of N_1. Then $f(\zeta_{P_H})$ be a Pythagorean Fuzzy HX Bi– Ideal of N_2, if ζ_{P_H}has a Supremum Property and ζ_{P_H} is a f-invariant.

PROOF:

Let H={< w, $\varrho_p(w)$, $\zeta_P(w)$ >/$w \in N$} be a Pythagorean Fuzzy Sets defined on a HX Near Ring $N1$. Then ζ_{P_H}={w, $\zeta_{P_\alpha}(w)$, $\zeta_{P_\beta}(w)$/$w \in N1$} be Pythagorean Fuzzy HX Bi – Ideal of a HX Near Ring $N1$.

Then $f(\zeta_{P_H}) = \{f(w), f(\zeta_{P_\alpha})(f(w)), f(\zeta_{P_\beta})(f(w))/w \in N1\}$ there exist $w, i, k \in N1$ such that $f(w)$, $f(i)$, $f(k) \in N2$

(i) $(f(\zeta_{P_\alpha}))(f(w) - f(i)) = (f(\zeta_{P_\alpha}))((w-i)) = \zeta_{P_\alpha}(w-i) \geqslant m\{\zeta_{P_\alpha}(w), \zeta_{P_\alpha}(i)\}$

Therefore

$(f(\zeta_{P_\alpha}))\,(f(w)-f(i)) \geqslant \; min\{f(\zeta_{P_\alpha})(f(w)),\; f(\zeta_{P_\alpha})(f(i))\}$

(ii) $(f\,(\zeta_{P_\alpha}))\,(f\,(w)\,f\,(i)\,(k)) = (f\,(\zeta_{P_\alpha}))\,((wik)) = \zeta_{P_\alpha}\,(wik) \geqslant \; min\{\zeta_{P_\alpha}(w), \zeta_{P_\alpha}(i), \zeta_{P_\alpha}(k)\}$

Therefore

$(f(\zeta_{P_\alpha}))\,(f(w)f(i)\,f(k)) \geqslant min\{f(\zeta_{P_\alpha})(f(w)),\; f(\zeta_{P_\alpha})(f(i)), f(\zeta_{P_\alpha})(f(k))\}$

(iii) $(f(\zeta_{P_\beta}))\,(f(w)-f(i)) = (f\,(\zeta_{P_\beta}))\,(f(w-i)) = \zeta_{P_\beta}\,(w-i) \leqslant \; max\{\,\zeta_{P_\beta}\,(w), \zeta_{P_\beta}\,(i)\}$

Therefore

$(f\,(\zeta_{P_\alpha}))\,(f\,(w)-f\,(i)) \leqslant \; max\,\{(\zeta_{P_\alpha})(f(w)),\; f(\zeta_{P_\alpha})(f(i))\}$

(iv) $(f\,(\zeta_{P_\beta}))\,(f\,(w)\,f\,(i)\,f\,(k)) = (f\,(\zeta_{P_\beta}))\,(f\,(wik)) = \zeta_{P_\beta}\,(wik) \leqslant \; max\{\,\zeta_{P_\beta}\,(w), \zeta_{P_\beta}\,(i), \zeta_{P_\beta}\,(k)\}$

Therefore

$(f(\zeta_{P_\beta}))\,(f(w)f(i)\,f(k)) \leqslant \; max\,\{f(\zeta_{P_\beta})(f(w)),\; f(\zeta_{P_\beta})(f(i)), f(\zeta_{P_\beta})(f(k))\}$

Hence

$f\,(\zeta_{P_Ƕ})$ is a Pythagorean Fuzzy HX bi – ideal of N_2.

THEOREM:C7

Let $Ƕ_p = \langle \varrho_{P_Ƕ}, \zeta_{P_Ƕ} \rangle$ be a Pythagorean Fuzzy HX Bi – Ideals of a HX Near Ring N and $f{:}\,[0,1] \to [0,1]$ be an increasing function then the Pythagorean Fuzzy Set $Ƕ_P^f = \langle \varrho_Ƕ^f, \zeta_Ƕ^f \rangle$ defined by $\varrho_{P_Ƕ}^{f(w)} = f\left(\varrho_{P_Ƕ}(w)\right)$ and $\zeta_{P_Ƕ}^{f(w)} = f\left(\zeta_{P_Ƕ}(w)\right)$ is a Pythagorean Fuzzy HX Bi – Ideal of N.

PROOF:

Let $Ƕ = \langle \varrho_{P_Ƕ}, \zeta_{P_Ƕ} \rangle$ be a Pythagorean Fuzzy HX Bi – Ideal of a HX Near Ring N for any $w, i, k \in N$

(i) $\varrho_{P_Ƕ}^f(w-i) = f\left[\varrho_{P_Ƕ}(w-i)\right] \geqslant f\left[min\left\{\varrho_{P_Ƕ}(w), \varrho_{P_Ƕ}(i)\right\}\right]$

$= min\left\{f\left(\varrho_{P_Ƕ}(w)\right), f\left(\varrho_{P_Ƕ}(i)\right)\right\}$

$= min\left\{\varrho_{P_Ƕ}^f(w), \varrho_{P_Ƕ}^f(i)\right\}$

Therefore

$\varrho_{P_Ƕ}^f(w-i) \geqslant \; min\left\{\varrho_{P_Ƕ}^f(w), \varrho_{P_Ƕ}^f(i)\right\}$

(ii) $\varrho_{P_Ƕ}^f(wik) = f\left[\varrho_{P_Ƕ}(wik)\right]$

$\geqslant f\left[min\left\{\varrho_{P_Ƕ}(w), \varrho_{P_Ƕ}(i), \varrho_{P_Ƕ}(k)\right\}\right]$

$= min\left\{f\left(\varrho_{P_Ƕ}(w)\right), f\left(\varrho_{P_Ƕ}(i)\right), f\left(\varrho_{P_Ƕ}(k)\right)\right\}$

$= min\{\mu_p{}^{Ƕf}(w), \mu_p{}^{Ƕf}(i), \mu_p{}^{Ƕf}(k)\}$

Therefore

$\varrho_{P_Ƕ}^f\,(w-i) \geqslant \; min\left\{\,\varrho_{P_Ƕ}^f(w), \varrho_{P_Ƕ}^f(i), \varrho_{P_Ƕ}^f(k)\right\}$

(iii) $\zeta_{P_Ƕ}^f(w-i) = f\left[\zeta_{P_Ƕ}(w-i)\right] \leqslant f\left[max\left\{\zeta_{P_Ƕ}(w), \zeta_{P_Ƕ}(i)\right\}\right]$

$= max\left\{f\left(\zeta_{P_Ƕ}(w)\right), f\left(\zeta_{P_Ƕ}(i)\right)\right\}$

$= max\left\{\zeta^f_{P_Ꟛ}(w), \zeta^f_{P_Ꟛ}(i)\right\}$

Therefore

$\zeta^f_{P_Ꟛ}(w-i) \preccurlyeq max\left\{\zeta^f_{P_Ꟛ}(w), \zeta^f_{P_Ꟛ}(i)\right\}$

(iv) $\zeta^f_{P_Ꟛ}(wik) = f\left[\zeta_{P_Ꟛ}(wik)\right]$

$\preccurlyeq f\left[max\left\{\zeta_{P_Ꟛ}(w), \zeta_{P_Ꟛ}(i),\right\}\zeta_{P_Ꟛ}(k)\right]$

$=max\left\{f\left(\zeta_{P_Ꟛ}(w)\right), f\left(\zeta_{P_Ꟛ}(i)\right), f\left(\zeta_{P_Ꟛ}(k)\right)\right\}$

$= max\left\{\zeta^f_{P_Ꟛ}(w), \zeta^f_{P_Ꟛ}(i), \zeta^f_{P_Ꟛ}(k)\right\}$

Therefore

$\zeta^f_{P_Ꟛ}(w-i) \preccurlyeq max\left\{\zeta^f_{P_Ꟛ}(w), \zeta^f_{P_Ꟛ}(i), \zeta^f_{P_Ꟛ}(k)\right\}$

THEOREM: C8

If a Pythagorean fuzzy set $Ꟛ =< \varrho_{P_Ꟛ}, \zeta_{P_Ꟛ} >$ satisfies the conditions

(i) $\varrho_{P_Ꟛ}(w-i) \succcurlyeq min\left\{\varrho_{P_Ꟛ}(w), \varrho_{P_Ꟛ}(i)\right\}$

(ii) $\zeta_{P_Ꟛ}(w-i) \preccurlyeq max\left\{\zeta_{P_Ꟛ}(w), \zeta_{P_Ꟛ}(i)\right\}$ then $\varrho_{P_Ꟛ}(0) \succcurlyeq \varrho_{P_Ꟛ}(w)$ and $\zeta_{P_Ꟛ}(0) \succcurlyeq \zeta_{P_Ꟛ}(w) \forall w \in N$

PROOF:

Let $w, i \in N$

$\varrho_{P_Ꟛ}(w-i) \succcurlyeq min\left\{\varrho_{P_Ꟛ}(w), \varrho_{P_Ꟛ}(i)\right\}$ $=min\left\{\varrho_{P_Ꟛ}(0), \varrho_{P_Ꟛ}(0)\right\}$ $\therefore \varrho_{P_Ꟛ}(w-i) \succcurlyeq \varrho_{P_Ꟛ}(0)$

$\zeta_{P_Ꟛ}(w-i) \preccurlyeq max\left\{\zeta_{P_Ꟛ}(w), \zeta_{P_Ꟛ}(i)\right\}$ $=max\left\{\zeta_{P_Ꟛ}(0), \zeta_{P_Ꟛ}(0)\right\}$ $\therefore \zeta_{P_Ꟛ}(w-i) \preccurlyeq \zeta_{P_Ꟛ}(0)$

$\therefore \varrho_{P_Ꟛ}(w-i) \succcurlyeq \varrho_{P_Ꟛ}(0)$ and $\zeta_{P_Ꟛ}(w-i) \preccurlyeq \zeta_{P_Ꟛ}(0). \forall w, i \in N$

THEOREM :C9

Let $Ꟛ =< \varrho_{P_Ꟛ}, \zeta_{P_Ꟛ} >$ be a Pythagorean Fuzzy HX bi – ideals of N then the sets $N_{Ꟛ_{\varrho P}} = \left\{w \in N / \varrho_{P_Ꟛ} = \varrho_{P_Ꟛ}(0)\right\}$ and $N_{Ꟛ_{\zeta P}} = \left\{w \in N / \zeta_{P_Ꟛ}(w) = \zeta_{P_Ꟛ}(0)\right\}$ are HX bi – ideals of N.

PROOF:

Let $w, i \in N_{Ꟛ_{\varrho P}}$ then $\varrho_{P_Ꟛ}(w) = \varrho_{P_Ꟛ}(0)$ and $\zeta_{P_Ꟛ}(i) = \zeta_{P_Ꟛ}(0)$ since $Ꟛ =< \varrho_{P_Ꟛ}, \zeta_{P_Ꟛ} >$ be a Pythagorean fuzzy HX bi – ideal of N we get $\varrho_{P_Ꟛ}(w-i) \succcurlyeq min\left\{\varrho_{P_Ꟛ}(w), \varrho_{P_Ꟛ}(i)\right\} = \varrho_{P_Ꟛ}(0)$ we get $\varrho_{P_Ꟛ}(w-i) = \varrho_{P_Ꟛ}(0)$ hence $w - i \in N_{Ꟛ_{\varrho P}}$ thus $N_{Ꟛ_{\varrho P}}$ is a subgroup of N.

Let $w, i, k \in N_{Ꟛ_{\varrho P}}$ then $\varrho_{P_Ꟛ}(w) = \varrho_{P_Ꟛ}(0)$, $\zeta_{P_Ꟛ}(i) = \zeta_{P_Ꟛ}(0)$ and $\zeta_{P_Ꟛ}(k) = \zeta_{P_Ꟛ}(0)$ Since $Ꟛ =< \varrho_{P_Ꟛ}, \zeta_{P_Ꟛ} >$ be a Pythagorean fuzzy HX bi – ideal of N we get

$\varrho_{P_Ꟛ}(wik) \succcurlyeq min\left\{\varrho_{P_Ꟛ}(w), \varrho_{P_Ꟛ}(i), \varrho_{P_Ꟛ}(k)\right\} = \varrho_{P_Ꟛ}(0)$ $\varrho_{P_Ꟛ}(w-i) = \varrho_{P_Ꟛ}(0)$

Hence

$wik \in N_{Ꟛ_{\varrho P}}$ Thus $N_{Ꟛ_{\varrho P}}$ is a Bi – Ideal of N.

$\zeta_{P_{ӈ}}(wik) \preccurlyeq max\left\{\zeta_{P_{ӈ}}(w), \zeta_{P_{ӈ}}(i), \zeta_{P_{ӈ}}(k)\right\} = \zeta_{P_{ӈ}}(0)\ \zeta_{P_{ӈ}}(w-i) = \zeta_{P_{ӈ}}(0)$

Hence $wik \in N_{ӈ_{\zeta_P}}$

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