

Theoretical Study of Isospin Mixing States with $T > 0$ in sd Even-Even $N=Z$ Nuclei Using Shell Model

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Abstract: Nuclear excited states with $T > 0$ in sd even-even $N=Z$ nuclei have been studied by using shell model. The calculations have employed the USDB Hamiltonian in order to predict the energy levels, the reduced electric quadrupole transition probabilities and reduced magnetic dipole transition probabilities. The study also include the average number of nucleons in each sd- active orbitals. The results compared with available experimental data. The comparison showed a good agreement between theoretical and experimental energy sates for most of the states studied in this work. On the other hand there was a difference between theoretical and experimental values of transition probabilities, but it can be said that it remained within the acceptable range of the difference.

Keywords: Shell Model, Isospin , OXBASH, USDB, $N=Z$ nuclei, energy levels

1. Introduction

Heisenberg was the first proposed the concept of isospin in 1932[1]. He introduced this quantum number in order to describe the charge independence of the nuclear force[2]. The connotation of charge independence and charge symmetry is formalized via the isospin. The charge dependent short range potential and electromagnetic interactions leads to break the isospin symmetry in nuclei, the isovector Coulomb interaction represents the most important part which mixes states separated by $\Delta T = 1$. The isospin mixing has an important connection with two nuclear phenomena, the spreading width of isobaric analog states and he superallowed Fermi β -decay [3].

Based on concepts of the shell model, the spin of the proton and neutron are both $t = 1/2$ with projection $t_z = -1/2$ and $t_z = +1/2$ for the proton and neutron, respectively. The eigenstates of many-body wave functions have $2T+1$ values of T_z going from $T_z = -T$ up to $T_z = +T$ in integer steps. For nucleus with Z protons and N neutrons, $T_z = (N - Z)/2$ with sign convention chosen, so that the more common neutron-rich nuclei will have positive T_z . The minimum value of T is $T_{min} = |T_z|$, and the maximum value is $T_{max} = A/2$. It is often that the lowest nuclear states have $T = T_z$ which refer to the two-nucleon system has is a stronger nuclear interaction with $T = 0$ than that in the $T = 1$ system[4], which explains that the isospin of the ground state is always equal T_{min} . Proton-neutron pairing is expected play an important role in the fundamental issue of nuclear structure. This pairing can contribute with additional binding energy which would be essential to their stability and the consequences of that about its location to the drip line.

Nuclei characterized by $N=Z$ received a remarkable interest over the resent years from the shell model calculations[5-8]. This interest shown from the studding of pairing correlations between fermions and the excitation states with isoscalar, $T=0$, and isovector, $T=1$. In $N=Z$ nuclei there are two channels in which proton neutron interaction can generate important pairing correlations. This correlations can be proton-neutron (pn) pairs correspond to $T=1$ and $T_z = 0$, and the other correlation is a components of the isospin $T=1$ are associated to the neutron-neutron (nn) and proton-proton (pp) pairs. The term mixed symmetry states indicates to all that nuclese states which have found with multiple isospin values[9]. Nuclei within sd- shells represent a good sample can be studied through the shell model. The characteristic of these nuclei is shown by the number of few valence particles, in addition to the presence of an effective interaction with good precision. USDB Hamiltonian had been introduced as an effective interaction by B. Alex Brown [10] which can be used to calculate the energy states for sd- nuclei with a good precision. In this work, the mixed isospin states with $T > 0$ have been calculated in even-even $N=Z$ nuclei within sd shell region by using shell model. The calculations have done for the energy values, and the probabilities of electric and magnetic transitions for these states. The aim of this work was to characterize the USDB precision of these states and according to the results, one can confirm the experimental results and present the distribution of valence particle for these states in sd shells.

2. Results and Discussions

The calculations were performed using the OXBASH code for Windows[11]. The code uses an M - scheme Slater determinant basis, a projection technique, and wave functions with good angular momentum J and isospin T , and this is the reason for choosing OXBASH code, since some other codes do not depend on T (like NuShellX[12]). The SD model spaces consisting of $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ shells above the ^{16}O nucleus and

the SDBA Hamiltonian defining the effective interaction with single-particle energies of 1.647, -3.164 , and -3.948 MeV for $0d_{3/2}$, $1s_{1/2}$, and $0d_{5/2}$, respectively[10].

The calculations have been carried out for the nuclei ^{20}Ne , ^{24}Mg , ^{28}Si and ^{36}Ar . Experimental and theoretical results are compared in Tables (1-4). All the energy levels has been adopted according to the ground states with $T=0$ for the intended nucleus. The experimental levels with “()” correspond to the cases in which the spin and/or parity of the corresponding states are not experimentally well established. The ground state of ^{20}Ne is a closed ^{16}O core with four nucleons distributed as two protons and two neutrons in *sd* space. From Table 1, a good agreements have shown for most energy levels with an acceptable difference for the two states $J=1+$ at energy 13.171 MeV and $J=0+$ at energy 13.640 MeV. A clear coincidence can be seen in the sequence of appearance of experimental and theoretical energy states.

Table 1: Experimental and theoretical energy levels for ^{20}Ne nucleus, the experimental data have taken from [13]. The sequence refers to the order of the state according to the theoretical results

Experimental			Theoretical			
Energy	$J\pi$	T	Energy (MeV)	$J\pi$	T	Sequence
10.273	2+	1	9.958	2+	1	1
10.884	3+	1	10.485	3+	1	1
11.090	4+	1	10.81	4+	1	1
12.221	2+	1	12.273	2+	1	2
13.171	1+	1	12.636	1+	1	1
13.484	1+	1	13.4613	1+	1	2
13.640	0+	1	14.431	0+	1	1
13.881	2+	1	13.827	2+	1	3
16.732	0+	2	16.84	0+	2	1
18.430	2+	2	18.587	2+	2	1

The predictions of USDB for the excited states of ^{24}Mg with $T=1$ show good agreement for all states with the experimental data as presented in Table 2. A first experimental state with $J=(4+)$, $T=1$, has not experimentally well established[13, 14], the theoretical results confirm the existence of this state especially if we know that the second theoretical state with $J=4+$, $T=1$ has found at energy 12.048 MeV. States with $J=1+$, and $J=2+$, $T=1,0$ at energies 9.828 and 12.403 MeV, respectively, have shown experimentally with mixture isospin value. According to the predictions of USDB, these state have $T=0$ with sequence 2 and 10, respectively, and this assumption is consistent with the experimental results.

Table 2: Experimental and theoretical energy levels for ^{24}Mg nucleus, the experimental data have taken from [13, 14]. The sequence refers to the order of the state according to the theoretical results.

Experimental			Theoretical			
Energy	$J\pi$	T	Energy	$J\pi$	T	Sequence
9.519	(4+)	1	9.399	4+	1	1
9.828	1+	0,1	9.789	1+	0	2
9.967	1+	1	9.93	1+	1	1
10.711	1+	1	10.723	1+	1	2
11.010	(3)	1	11.202	3+	1	2
12.049	4+	1	12.049	4+	1	2
12.403	2+	0,1	12.466	2+	0	10
12.636	4+	1	12.4	4+	1	3
12.954	1+,2+	1	13.003	2+	1	5
			13.02	1+	1	4
13.044	0+	0,1	12.926	0+	1	1
13.343	3+	0,1	13.243	3+	1	6
			13.323	3+	0	8
15.436	0+	(2)	15.379	0+	2	1

Energy level with $J=(3)$, $T=1$ at 11.010 MeV appeared experimentally with uncertainty spin and parity. The theoretical results given first sequence $J=3+$, $T=1$ at energy 10.737 MeV, and second $J=3+$, $T=1$ at energy 11.202 MeV. It is likely that the second state is the closest to being equivalent to the experimental value. The doublet consisting of the state $J=1+,2+$, $T=1$ is present amongst the theoretically predicted states as two states with very close energy values. States with $J=0+$ and $J=3+$ with $T=1,0$ at energies 13.044 and 13.343 MeV, respectively,

have shown experimentally with mixture isospin value. According to the predictions of USDB, these state have T=1 with sequence 1 and 6, respectively. The uncertain isospin state with T=2 has confirmed theoretically with good agreement with the experimental value of the energy.

Nucleus 28Si has 12 nucleons distributed as 6 protons and 6 neutrons in sd space. The comparison between experimental and theoretical states has presented in Table 3. Many experimental states are not experimentally well established as we can find in references[13, 15].

Table 3: Experimental and theoretical energy levels for 28Si nucleus, the experimental data have taken from[13, 15]. The sequence refers to the order of the state according to the theoretical results.

Experimental			Theoretical			
Energy (MeV)	J ^π	T	Energy (MeV)	J ^π	T	Sequence
9.315	3 ⁺	1	9.417	3 ⁺	1	1
9.381	2 ⁺	1	9.430	2 ⁺	1	1
10.272	0 ⁺	1	10.403	0 ⁺	1	1
10.376	(3 ⁺ ,4 ⁺)	1	10.564	3 ⁺	1	2
10.883	(2,3) ⁺	1	10.935	2 ⁺	1	2
10.900	(1 ⁺)	1	10.91	1 ⁺	1	2
11.432	(2 ⁺)	0,1	11.489	2 ⁺	1	3
11.446	(1 ⁺)	1	11.482	1 ⁺	1	3
11.778	(2 ⁺)	0,1	11.833	2 ⁺	1	4
11.867	(4 ⁺)	1	11.945	4 ⁺	1	2
11.899	4 ⁺	0,1	12.012	4 ⁺	0	8
12.331	(1 ⁺)	1	12.422	1 ⁺	1	4
12.573	(2 ⁺)	1	12.671	2 ⁺	1	5
12.754	(1,2) ⁺	0,1				
12.855	(4 ⁺)	0,1	12.885	4 ⁺	1	3
12.917	(2,3) ⁺	1	12.895	2 ⁺	1	6
12.917	(3 ⁺)	1	12.898	3 ⁺	1	5
13.039	(0 ⁺)	1	13.083	0 ⁺	1	3
13.188	(2 ⁺)	0,1	13.208	2 ⁺	1	7
13.320	(1 ⁺)	1	13.318	1 ⁺	1	6
13.425	(5 ⁺)	1	13.664	5 ⁺	1	2
13.483	(2 ⁺)	0,1	13.548	2 ⁺	1	8
13.557	(5 ⁺ ,4 ⁺)	0,1	13.592	5 ⁺	0	6
13.979	(4 ⁺)	1	13.922	4 ⁺	1	5
15.227	(0 ⁺)	2	15.361	0 ⁺	2	1

Based on the agreement between the experimental and theoretical results for the first three states with J=3+,2+ and 0+, the predictions of USDB give a clear declaration for the next energy levels. The doublet consisting of the states J=(3+,4+), (2,3)+ and (5+,4+) at energies 10.376, 10.883 and 13.557 MeV, respectively, are found theoretical with a single J values. These J values are J=3+,2+ and 5+ at energies 10.564, 10.935 and 13.592 MeV, respectively. Theoretically, the last states with J=5+ suggest it possesses isospin T=0 and it is found has 5th sequence. Identification of these states has done based on the spin sequences and energy gaps and state-to-state correspondences. The experimental spectrum of 28Si nucleus show many single J state with mixture isospin value. All these states have found theoretically with isospin T=1, except for J=4+ at energy 11.899 MeV which seemed more compatible with isospin T=0 and 8th sequence. The theoretical results did not predict any equivalent state for J= (1,2)+ at energy 12.754 MeV.

Table 4: Experimental and theoretical energy levels for 36Ar nucleus, the experimental data have taken from[13, 16]. The sequence refers to the order of the state according to the theoretical results.

Experimental			Theoretical			
Energy (MeV)	J ^π	T	Energy (MeV)	J ^π	T	Sequence
6.614	2 ⁺	1	6.375	2 ⁺	1	1
7.336	3 ⁺	1	7.211	3 ⁺	1	1
7.710	1 ⁺	1	7.446	1 ⁺	1	1
8.131	1 ⁺	1	8.107	1 ⁺	1	2
8.556	2 ⁺	1	8.394	2 ⁺	1	2

9.700 0⁺ 1 9.557 0⁺ 1 1

Table 4 shows the experimental and theoretical states for ³⁶Ar nucleus. It is evident from the Table 4 that the theoretical calculations for ³⁶Ar nucleus were remarkably consistent with the experimental values. This can be explained by the fact that all states that possess $T > 0$ appeared experimentally with certain J and T values.

Figure 1 shows the average number of nucleons in each *sd*- active orbitals for protons and neutrons that used in calculated the first J^+ states for the nuclei studied in this work. For ²⁰Ne, shells *0d*_{5/2} and *1s*_{1/2} have the largest contributor in configuration the states with $T > 0$, and this can be seen through the change in the average number of particles per shell. The contribution of *1s*_{1/2} has decreases in ²⁴Mg configurations with a majority for *0d*_{5/2} shell. It is can be explain that as a result of increasing the valance particles which lades to fill the *0d*_{5/2} first in order for more stability for the nucleus. This behavior continues to appear when progressing to the nuclei with higher mass numbers as in ²⁸Si and ³⁶Ar. The filling of *0d*_{5/2} shell is accompanied by a decrease in the number of particles that occupy the *1s*_{1/2} and *0d*_{3/2} shells until we reach ³⁶Ar. With 20 valance particles, ³⁶Ar, *0d*_{5/2} shell becomes completely full and this leads to increase the contribution of *1s*_{1/2} and *0d*_{3/2} shells in configure states.

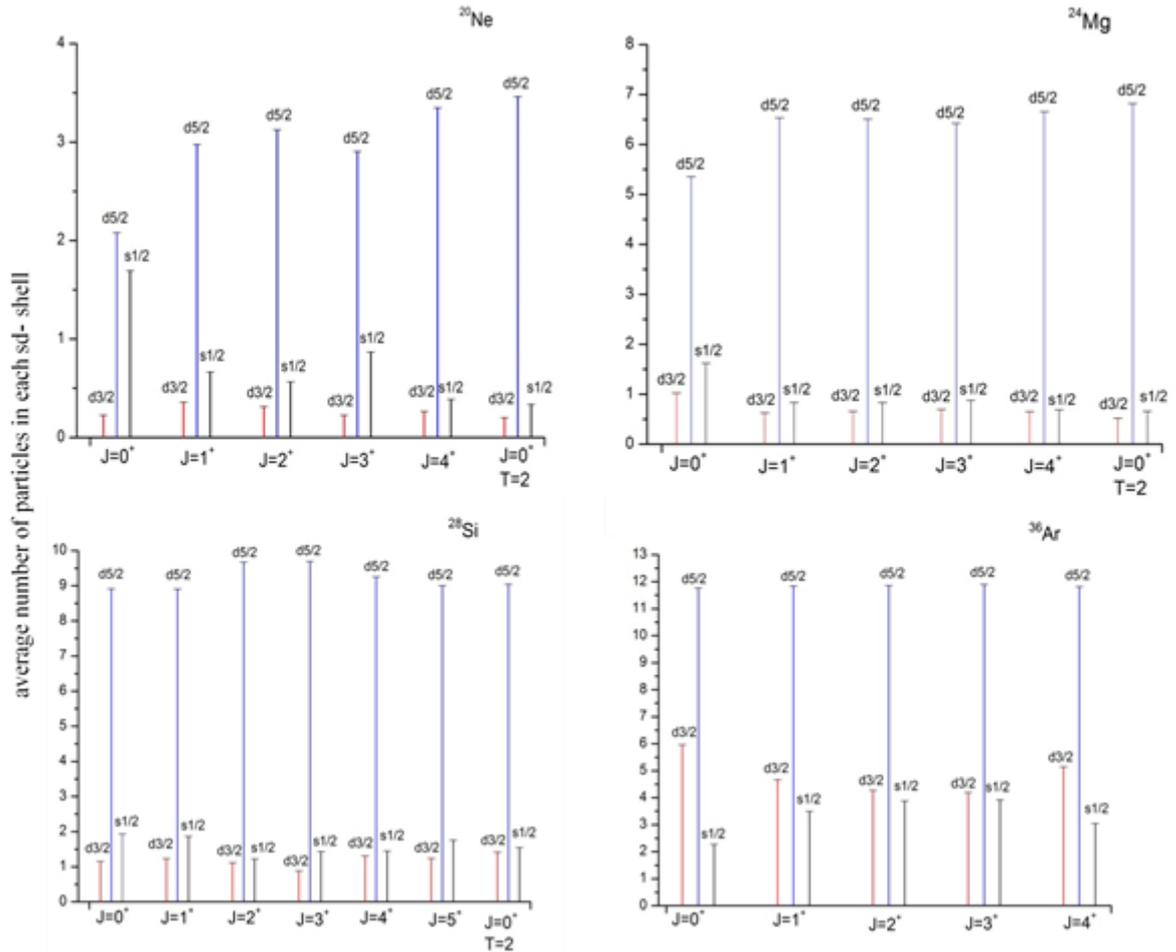


Figure 1: (Color online) The average number of particles in each *sd*- shell for the first J^+ states with $T=1$ and 2 in nuclei ²⁰Ne, ²⁴Mg, ²⁸Si and ³⁶Ar.

In shell model calculation, the reduced electric quadrupole transition probabilities $B(E2)$ and reduced magnetic dipole transition probabilities $B(M1)$ represent a valuable factor in determining the efficiency of the predicted wave function. Isospin symmetry plays an important role to predict transition probabilities in the even-even nuclei. For this purpose, the $B(E2)$ and $B(M1)$ have calculated for all available experimental values for the nuclei under consideration.

For $B(E2)$ values, comparison of theory with experiment is shown in Table 5. By using default values of the effective charges for proton and neutron $e_p = 1.36(5)$ and $e_n = 0.45(5)$ [17, 18], respectively, a clear difference appear between the experimental and theoretical values. The calculated $B(E2)$ values appeared with higher values, but in general, it can be said that the predicted values remained within the range of acceptable comparison. The same behavior can also be observed with $B(M1)$ values as shown in Table 6. This difference can be interpreted based on the default values of effective charges, which are used in calculated of $B(E2)$ value, and g factors, which are used in calculated of $B(M1)$ value, were found by calibration of the transitions between levels bearing the same isospin as we can see in reference [18]. A previous theoretical researches have proven that there are

important effects of effective charges and g factors values that must be taken into account when calculating electric and magnetic transition[19, 20].

Table 5: Theoretical and experimental B(E2) values for ²⁰Ne, ²⁸Si and ³⁶Ar. Experimental data taken from[13-16]. The theoretical states have given with their sequences according on the closest energy levels

Nucleus	Experimental							Theoretical				
	E_i (MeV)	J_{in}	T_{in}	E_f (MeV)	J_{fin}	T_{fin}	$B(E2)$ $e^2 m_f^4$	$J_{i in}^+$	T_{in}	$J_{i fin}^+$	T_{fin}	$B(E2)$ $e^2 m_f^4$
	²⁰ Ne	11.090	4 ⁺	1	1.633	2 ⁺	0	0.032	4 ₁	1	2 ₁ ⁺	0
²⁸ Si	9.315	3 ⁺	1	6.276	3 ⁺	0	0.686	3 ₁	1	3 ₁	0	1.37
	9.381	2 ⁺	1	1.779	2 ⁺	0	0.151	2 ₁	1	2 ₁	0	0.371
				0.00	0 ⁺	0	0.232			0 ₁	0	0.882
	10.272	0 ⁺	1	1.779	2 ⁺	0	> 0.181	0 ₁	1	2 ₁	0	0.414
³⁶ Ar	7.336	3 ⁺	1	4.414	4 ⁺	0	0.010	3 ₁	1	4 ₁	0	0.169

Table 6: Theoretical and experimental B(M1) values for ²⁰Ne, ²⁸Si and ³⁶Ar. Experimental data taken from[13-16]. The theoretical states have given with their sequences according on the closest energy levels

Nucleus	Experimental							Theoretical				
	E_i (MeV)	J_{in}	T_{in}	E_f (MeV)	J_{fin}	T_{fin}	$B(M1)$ μ^2	$J_{i in}^+$	T_{in}	$J_{i fin}^+$	T_{fin}	$B(M1)$ μ^2
	²⁰ Ne	10.273	2 ⁺	1	7.833	2 ⁺	0	0.046	2 ₁	1	2 ₂	0
	18.430	2 ⁺	2	12.221	2 ⁺	1	0.107	2 ₁	2	2 ₂	1	0.809
²⁸ Si	9.315	3 ⁺	1	6.276	3 ⁺	0	5.37	3 ₁	1	3 ₁	0	1.305
	9.381	2 ⁺	1	8.258	2 ⁺	0	0.143	2 ₁	1	4 ₂	0	0.925
				6.276	3 ⁺	0	0.322			3 ₁	0	0.117
			1.779	2 ⁺	0	0.035			2 ₁	0	0.145	
	10.900	(1 ⁺)	1	0.00	0 ⁺	0	0.238	1 ₂	1	0 ⁺	0	0.415
	11.446	(1 ⁺)	1	0	0 ⁺	0	1.51	1 ₃	1	0 ⁺	0	1.164
³⁶ Ar	7.336	3 ⁺	1	4.414	4 ⁺	0	0.002	3 ₁	1	4 ₁	0	0.07

3. Conclusion:

In this study, the mixed isospin states with $T > 0$ for nuclei ²⁰Ne, ²⁴Mg, ²⁸Si and ³⁶Ar have been calculated by employed the USDB sd-shell Hamiltonian. The calculations include energy states, reduced electric quadrupole transition probabilities and reduced magnetic dipole transition probabilities. All the predictions results have been compared with a viable experimental data. The USDB calculations of the energy levels show significant agreement with the experimental states for nucleus ²⁰Ne. The theoretical work has confirmed the existence of the states $J=(4^+)$ and (3) in ²⁴Mg spectrum. On the other hand, the experimental results of the ²⁸Si nucleus revealed a number of uncertain levels. From this work, a number of them were confirmed, in addition to determining the isospin values for them, as they appeared experimentally with mixture isospin values. Also some special cases

present in this nucleus were discussed. The comparison between experimental and theoretical values showed good agreement regarding the energy levels of the nucleus ^{36}Ar .

Shell model provide the capability of know the distribution of valence nucleons in active orbitals. In this work, the distribution of valence nucleons has studied for configure the target states in this work. The study showed the important shells that contributed to the formation of the wave functions of the states, which differed according to the nucleus. Finally, the results of the calculations of $B(E2)$ and $B(M1)$ appeared not identical with experimental data, but overall it have stayed within the range of experimental values.

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