Research Article

Robustness Of The Standard Intuitionistic Fuzzy Sets For Image Enhancement

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Abstract : This paper centered the robustness of the standard intuitionistic fuzzy sets which are recognized in the fuzzy as well as non-fuzzy regions of $\max - \min$ intervals. The max-min and max-product composition of intuitionistic fuzzy relation (IFR) is discussed through intuitionistic fuzzy set (IFS). Each set changes its position with adaptive speed in a stochastic manner. Simultaneously index of fuzziness is measured by the amount of vagueness of IFS and classified according to max, min function. We extend these IFS to digital image enhancement techniques to obtain accurate information from the midst of vague images. Type I fuzzy set, establishes the region of vague domains and range of images with its lower and upper membership degrees. Based on the lower and higher membership values, the discomposed membership degrees have fuzzy weighted average which represents the output image. The new output is imported into the main process of the fuzzy sets in order to find the adaptive and the best parameter according to its maximum and minimum level. Concurrently used to find the disparity between IFS is measured to optimize the decision making sense.

Keywords: Intuitionistic Fuzzy Sets, image enhancement, decision making system, knowledge based predictions.

I Introduction

Researchers have made a significant effort in recent years to demonstrate IFS (Intuitionistic Fuzzy Sets) efficacy in ambiguous circumstances, ambiguity modelling problems and its applicability in a wide range of fields, such as decision-making, fuzzy optimization, pattern recognition, medical diagnosis, etc. Uncertainty in medical diagnosis is varies from person to person, at the physical as well as at the mental level, the symptoms of illness communicated by patients are verbal in nature. The physician shall assess the possible disease suffered by patients on the basis of the symptoms stated by the patients.

The major, as well as minor, signs of almost all diseases are reported from current research and expertise in medical science. In the initial phase of any illness, the patient's symptoms are closely examined and the patient may benefit from the disease by contrasting the symptoms one might indicate.

Initially, Lotfi A Zadeh introduced and presented the idea of Fuzzy Set Theory in 1965 [13]. Later, in 1986, Krassimir Atanassov developed the IFS method and extend the same in the classical fuzzy set theory [1, 2] and I. Couso and H. Bustince, further developed the IFS in new direction [4]. However in the assumption of IFS, a component in the universal set is delegated to each component of a degree of membership, a degree of non-membership and a degree of hesitation. This is one of the key cause that why IFS has been regarded as a more efficient and productive method than the Fuzzy Set Theory.

Similarity is an important method that can be used in decision-making problems to overcome ambiguity by IFS theory.A variety of similarity tests have been implemented on the IFS by several researchers. IFS multi-attribute decision making method was discussed by S.-M. Chen and W.H. Han [3].Then Şahin improved a comparison measure and added few more illustrations to enrich their results in IFS [8]. Y. Yang, et al [11] , extended the distance between Hausdorff in 2004 and introduced three new similarity measures for IFSs. Using the cosine function, E. Szmidt et al introduced a new weighted similarity measure in 2001 [9,10]. In 2018, S. Wan, et al, developed the concepts in consistency of interval-valued intuitionistic fuzzy analysis [13]. In 2007, Xu developed multi-criteria concepts in the decision making problems. Generalized intuitionistic fuzzy sets applications are developed by E. B. Jamkhaneh et al 2018 [5]. A clustering method for multi-stage hesitant fuzzy linguistic terms was extended by Z. Ma et al [6]. A general form of similarity measure was developed by S. Wan, F. Wang [12], connecting the two standard parameters and the degree of uncertainty . R. T. Ngan et al developed an axiomatic approach to IFS similar and dissimilar measurement [7].

This article is classified as follows: Section II dealt with the implementation of IFS through its membership and non-membership functions are showed. In section III, we proposed a technique of intuitionistic fuzzy set for image enhancement through the transformation of fuzzy *G*-singleton set using a specific membership and non membership function. Section IV gives detailed insight of IFS Strategy in Medical Diagnosis and decision making on disparity of IFS. Finally section V concludes the paper.

II Implementation of IFS

An IFS A_i in E_i (fixed set) is,

$$A_i = \{< x, \mu_{A_i}(x), v_{A_i}(x) > | \ x \in E_i\}$$

here $\mu_{A_i} : E_i \to [0,1]$ and $v_{A_i} : E_i \to [0,1]$ describes the degree of membership and degree of non-membership function and $x \in E_i$ to the set A, which is a subset of E_i, and

$$0 \leq \mu_{A_i}(x) + v_{A_i}(x) \leq 1$$

The value of $\pi_{A_i}(x)$ is defined as $\pi_{A_i}(x) = 1 - (\mu_{A_i}(x) - v_{A_i}(x))$ is called the hesitation element.

`**Definition 2.1:** If A_i and B_i are two independent IFS and belongs to the set E_i, then

$A_i \subset B_i$ iff $\forall\, x \in E_i, [\mu_{A_i}(x) \leq \mu_{B_i}(x) and\ v_{A_i}(x) \geq v_{B_i}(x)]$

$A_i = B_i$ iff $\forall\, x \in E_i, [\mu_{A_i}(x) = \mu_{B_i}(x) and\ v_{A_i}(x) = v_{B_i}(x)]$

$\overline{A_i} = \{< x, v_{A_i}(x), \mu_{A_i}(x) > | \ x \in E_i\}$

$A_i \cap B_i = \{< x, \min\left(\mu_{A_i}(x), \mu_{B_i}(x)\right), \max\left(v_{A_i}(x), v_{B_i}(x)\right) > | \ x \in E_i\}$

$A_i \cup B_i = \{< x, \max\left(\mu_{A_i}(x), \mu_{B_i}(x)\right), \min\left(v_{A_i}(x), v_{B_i}(x)\right) > | \ x \in E_i\}$

Definition 2.2: : Let A_i (X →Y) and R (Y →Z) be two IFR. The max-min-max composition RoA_i is the intuitionistic fuzzy relation from X to Z, defined by the membership function

$$\mu_{R_i \circ A_i}(x,z) = \bigvee_y [\mu_{A_i}(x,y) \wedge \mu_{R_i}(y,z)]$$

and its non-membership function as

$$v_{R_i \circ A_i}(x,z) = \bigwedge_y \left[v_{A_i}(x,y) \vee v_{R_i}(y,z)\right], \forall (x,z) \in X \times Z \ and \ \forall\, y \in Y$$

(here $\vee = max$, $\wedge = min$).

An axiomatic approach of similar concepts of applied in intuitionistic fuzzy sets. One of the most popular optimization procedure in intuitionistic fuzzy sets are each set changes its position with adaptive speed in a stochastic manner. Suppose that N_1 is the intuitionistic fuzzy set within the dimensional search space M_1. Then for each instant, the position of the k^{th} element is defined as $X_k(x_{k_1}, x_{k_2}, \dots, x_{k_{M_1}})$, which disclose a feasible solution to the optimization problem. Here the current position of each set symbolize as $v_k(v_{k_1}, v_{k_2}, \dots, v_{k_{M_1}})$ and $x_k(x_{k_1}, x_{k_2}, \dots, x_{k_{M_1}})$ be the relative variation of the set $v_k(v_{k_1}, v_{k_2}, \dots, v_{k_{M_1}})$. The difference between $v_k(v_{k_1}, v_{k_2}, \dots, v_{k_{M_1}})$ and $x_k(x_{k_1}, x_{k_2}, \dots, x_{k_{M_1}})$ is presented in $\omega_g(\omega_{g_1}, \omega_{g_2}, \dots, \omega_{g_{M_1}})$, p_{km} is the past position of the form. The last position indicated by the entire form is p_{lm} . The elements are treated with the following equations:

$$v_{km}^{i+1} = \omega^i * v_{km}^i + c_1 * (\omega_g) * (p_{km} - x_{km}^i)\Delta t + c_2 * (\omega_g) * (p_{lm} - x_{km}^i)\Delta t$$

$$x_{km}^{i+1} = x_{km}^i + \Delta t * v_{km}^i / k_{max} \tag{1}$$

$$\omega^k = \omega_{max} - k * (\omega_{max} - \omega_{min}) \tag{2}$$

and c be the coefficient, the random function hold the uniform distribution $U(0,1)$ and Δt represents the difference factor and unit of time. In addition to that v_{km}^{i+1} and x_{km}^{i+1} must be in limited conditions as follows:

$$v_{km}^{i+1} = \begin{cases} v_{km}^{i+1} - v_{max} \leq v_{km}^{i+1} \leq v_{max} \\ \quad v_{max} \ \ v_{km}^{i+1} > v_{max} \\ \ -v_{max} \ \ v_{km}^{i+1} > -v_{max} \end{cases} \tag{3}$$

$$x_{km}^{i+1} = \begin{cases} x_{km}^{i+1} - x_{max} \leq x_{km}^{i+1} \leq x_{max} \\ \quad x_{max} \ \ x_{km}^{i+1} > x_{max} \\ \ -x_{max} \ \ x_{km}^{i+1} > -x_{max} \end{cases} \tag{4}$$

$$x_{nit}^{i+1} = x_{min} + (\omega_g) * (x_{max} - x_{min}) \tag{5}$$

The high and updated number of iterations is indicated by (3) to (5) respectively.

IIII Proposed intuitionistic fuzzy set for image enhancement

In fuzzy domain, image enhancement techniques dealt with the vagueness and uncertainty of images. Suppose that X is an $M_1 \times N_1$ image with dynamic gray levels L ranging from L_{min} to L_{max} respectively, and x_{ij} represents the associated gray pixel level. Now, X_1 is transformed into a fuzzy G-singleton set using a specific membership function as

$$G = \{\mu_{X_1}(x_{ij}) \ i = 1,2, \dots, N_1\} \quad (6)$$

where $0 \leq \mu_{X_1}(x_{ij}) \leq 1$ and $\mu_{X_1}(x_{ij})$ represent the degree of basic image properties, such as brightness, gray, etc., as a function of the $(i,j)^{th}$ pixel.

In general, the standard intuitionistic fuzzy set S is applied into the membership function is classified like:

$$S = \begin{cases} 0 & x < p \\ 2 \times \left(\frac{x-p}{r-p}\right)^2 & p \leq x < q \\ 1 - 2 \times \left(\frac{x-p}{r-p}\right)^2 & q \leq x < r \\ 1 & x \geq r \end{cases} \quad (7)$$

where p, q and r are fuzzy parameters. Assume p as the point of intersection and define it as $\frac{q+r}{2}$. In a analogous way we define q as $\frac{p+r}{2}$ and r as $\frac{q+p}{2}$. Now calculate the width of the entire region according to $2\Delta p = rq$, $2\Delta q = rp$ and $2\Delta r = qp$ its corresponding non-fuzzy regions intervals are $[L_{min}, p]$, $[L_{min}, q]$, $[L_{min}, r]$ and $[p, L_{max}]$, $[q, L_{max}]$ and $[r, L_{max}]$.

Index of fuzziness is measured according to max, min function as,

$$\gamma(X_1) = \sum_{k=0}^{L-1} max[\mu_X(k), 1 - \mu_X(k)]\, g(k)$$

$$\gamma(X_2) = \sum_{k=0}^{L-1} min[\mu_X(k), 1 - \mu_X(k)]\, g(k) \quad (8)$$

and $g(k)$ referred the intensity level k. Traditionally, a fuzzy set is largely associated with an indefinite attribution of degree of belonging and is therefore more coherent than its preceding fuzzy set. In general, $g(k) \in \tilde{G}$, the region of uncertainty is measured from

$$\tilde{G} = \{(|x, \mu_L(x), \mu_U(x)|) , \quad \forall\, x \in X_1 \quad (9)$$

The lower and upper membership values are:

$$\mu_{L_1}(x) = \left[\mu(x)^{\frac{q}{2}}\right], and\ \mu_{U_1}(x) = [\mu(x)^{q*2}] \quad (10)$$

where $x \in [0, 1]$. The decomposed membership degrees consist of a fuzzy weighted average representing the output image, based on the lower and higher membership values are represented by μ_{g_e}. The output image μ_{g_e} is articulated as,

$$\mu_{g_e}(x) = \left(\eta \times \mu_{L_1}(x) + (1-\eta) \times \mu_{U_1}(x)\right) .\gamma(X_1) \quad (11)$$

where η is a best approximated parameter and is defined with converging factors.

IV IFS Strategy in Medical Diagnosis and designing the fitness functions based on IFS

The position of elements in the set be defined as $X_k(x_{k_1}, x_{k_2}, \dots, x_{k_M})$. Its indicates the possible optimal explanation and solution to the problem with the dimension of the set $M_1 \times N_1$. The new intuitionistic fuzzy set's output is, in general imported into the main process of the fuzzy sets $\mu_{L_1}(x)$ and $\mu_{U_1}(x)$ in order to adaptively find the best parameter according to its maximum and minimum.

Once the best parameter is identified, then consider the position matrix X_s, and its corresponding output position matrix V_s as follows:

$$X_s = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1M} \\ x_{21} & x_{22} & \cdots & x_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ x_{N1} & x_{N1} & \cdots & x_{NM} \end{pmatrix} \quad and \quad V_s = \begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1M} \\ v_{21} & v_{22} & \cdots & v_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ v_{N1} & v_{N1} & \cdots & v_{NM} \end{pmatrix} \qquad (12)$$

Finally, the processed image is achieved and given as,

$$G_e(i,j) = \bigcup_{i=1}^{M_1} \bigcup_{j=1}^{N_1} \mu_{A_i}(i,j) * (L-1) * (U-1) + \mu_{A_i}(\eta_i) \qquad (13)$$

Definition 4.1: Consider two IFS named as $A_i = \{< \mu_{A_i}(\eta_i), v_{A_i}(\eta_i) >\}; \eta_i \in X$ and $B_i = \{< \mu_{B_i}(\eta_i), v_{B_i}(\eta_i) >\}; \eta_i \in X$ in the finite set $X = \eta_1, \eta_2, \ . \ . \ . \ \eta_n$, then its maximum values are given by

$$S_{ma}(A_i, B_i) = \Big[\Big\{\max\big(\mu_{A_i}(\eta_i), \mu_{B_i}(\eta_i)\big) + \max\big(v_{A_i}(\eta_i), v_{B_i}(\eta_i)\big) + \max\big(\pi_{A_i}(\eta_i), \pi_{B_i}(\eta_i)\big)\Big\}$$
$$- \Big\{\Big(\frac{\mu_{A_i}(\eta_i) + \mu_{B_i}(\eta_i)}{2}\Big) + \Big(\frac{v_{A_i}(\eta_i) + v_{B_i}(\eta_i)}{2}\Big) + \Big(\frac{\pi_{A_i}(\eta_i) + \pi_{B_i}(\eta_i)}{2}\Big)\Big\}\Big]$$
$$= \sum_{i=1}^{n} \left[\upsilon \begin{pmatrix} \mu_{A_i}(\eta_i) \\ \mu_{B_i}(\eta_i) \end{pmatrix} + \upsilon \begin{pmatrix} v_{A_i}(\eta_i) \\ v_{B_i}(\eta_i) \end{pmatrix} + \upsilon \begin{pmatrix} \pi_{A_i}(\eta_i) \\ \pi_{B_i}(\eta_i) \end{pmatrix} \right]$$

Here, $\upsilon\begin{pmatrix} x \\ y \end{pmatrix} = \max(x,y) - \left(\frac{x+y}{2}\right)$ for all x and y. Then $S_{ma}(A_i, B_i)$ satisfied the following properties:

(i) $S_{ma}(A_i, B_i) \geq 0$, when the sets A_i and B_i are identical then the equality hold.
(ii) $S_{ma}(A_i, B_i) = S_{ma}(B_i, A_i)$
(iii) $S_{ma}(A_i, B_i) = S_{ma}(A_i{}^c, B_i{}^c)$
(iv) $S_{ma}(A_i, A_i{}^c) = 0 \Leftrightarrow \mu_{A_i}(x_i) = v_{A_i}(x_i)$ for all possible values of X.
(v) $S_{ma}(A_i, B_i{}^c) = S_{ma}(A_i{}^c, B_i)$

Proof. (i) The inequality related to $\max(\alpha, \beta) \geq \left(\frac{\alpha+\beta}{2}\right)$. Hence, the proposed measure given by $S_{ma}(A_i, B_i)$ is obviously positive for any of its parameters within the region.

(ii) The proof is obvious.

(iii) To prove $S_{ma}(A_i, B_i) = S_{ma}(A_i{}^c, B_i{}^c)$, consider

$$S_{ma}(A_i, B_i) = \sum_{i=1}^{n} \left[\upsilon \begin{pmatrix} \mu_{A_i}(\eta_i) \\ \mu_{B_i}(\eta_i) \end{pmatrix} + \upsilon \begin{pmatrix} v_{A_i}(\eta_i) \\ v_{B_i}(\eta_i) \end{pmatrix} + \upsilon \begin{pmatrix} \pi_{A_i}(\eta_i) \\ \pi_{B_i}(\eta_i) \end{pmatrix} \right]$$
$$S_{ma}(A_i, B_i) = \sum_{i=1}^{n} \left[\upsilon \begin{pmatrix} v_{A_i}(\eta_i) \\ v_{B_i}(\eta_i) \end{pmatrix} + \upsilon \begin{pmatrix} \mu_{A_i}(\eta_i) \\ \mu_{B_i}(\eta_i) \end{pmatrix} + \upsilon \begin{pmatrix} \pi_{A_i}(\eta_i) \\ \pi_{B_i}(\eta_i) \end{pmatrix} \right]$$
$$S_{ma}(A_i, B_i) = S_{ma}(A_i{}^c, B_i{}^c)$$

(iv) Consider $S_{ma}(A_i, A_i{}^c) = 0$

$$\Leftrightarrow \sum_{i=1}^{n} \left[\upsilon \begin{pmatrix} \mu_{A_i}(\eta_i) \\ \mu_{B_i}(\eta_i) \end{pmatrix} + \upsilon \begin{pmatrix} v_{A_i}(\eta_i) \\ v_{B_i}(\eta_i) \end{pmatrix} + \upsilon \begin{pmatrix} \pi_{A_i}(\eta_i) \\ \pi_{B_i}(\eta_i) \end{pmatrix} \right] = 0$$
$$\Leftrightarrow \sum_{i=1}^{n} \Big[\Big\{\max\big(\mu_{A_i}(\eta_i), v_{A_i}(\eta_i)\big) + \max\big(v_{A_i}(\eta_i), \mu_{A_i}(\eta_i)\big) + 1 - \mu_{A_i}(x_i) - v_{A_i}(x_i)\Big\}$$
$$- \{\mu_{A_i}(\eta_i) + v_{A_i}(\eta_i) + \pi_{A_i}(\eta_i)\}\Big] = 0$$
$$\Leftrightarrow \sum_{i=1}^{n} \Big(\Big(\max\big(\mu_{A_i}(\eta_i), v_{A_i}(\eta_i)\big) - \mu_{A_i}(x_i)\Big) + \max\big(\mu_{A_i}(\eta_i), v_{A_i}(\eta_i)\big) - v_{A_i}(x_i)\Big) = 0$$

$\Leftrightarrow \mu_{A_i}(x_i) = v_{A_i}(x_i)$ for all possible values of X.

(v) Consider,

$$S_{ma}(A_i, B_i{}^c) = \sum_{i=1}^{n} \left[\upsilon \begin{pmatrix} \mu_{A_i}(\eta_i) \\ v_{B_i}(\eta_i) \end{pmatrix} + \upsilon \begin{pmatrix} v_{A_i}(\eta_i) \\ \mu_{B_i}(\eta_i) \end{pmatrix} + \upsilon \begin{pmatrix} 1 - \mu_{A_i}(\eta_i) - v_{B_i}(\eta_i) \\ 1 - v_{A_i}(\eta_i) - \mu_{B_i}(\eta_i) \end{pmatrix} \right]$$

$$S_{ma}(A_i, B_i{}^c) = \sum_{i=1}^{n}\left[\upsilon\begin{pmatrix} v_{A_i}(\eta_i) \\ \mu_{B_i}(\eta_i) \end{pmatrix} + \upsilon\begin{pmatrix} \mu_{A_i}(\eta_i) \\ v_{B_i}(\eta_i) \end{pmatrix} + \upsilon\begin{pmatrix} 1-\mu_{A_i}(\eta_i) - v_{B_i}(\eta_i) \\ 1-v_{A_i}(\eta_i) - \mu_{B_i}(\eta_i) \end{pmatrix}\right]$$
$$S_{ma}(A_i, B_i{}^c) = S_{ma}(A_i{}^c, B_i)$$

Dissimilarity measures are obtained from max min functions. These results can be implemented in the digital image processing to enhance the image. This result reduced the risk attitudes of the fuzzy knowledge based decision makers.

V Conclusion

The new intuitionistic fuzzy set is imported into the fuzzy sets $\mu_{L_1}(x)$ and $\mu_{U_1}(x)$ addressed the limitations of upper and lower values of membership and non membership function in order to adaptively find the best parameter according to its maximum and minimum are defined in this paper. The disparity between IFS is measured to optimize the knowledge based decision making of any digital image with various ranges and domains arrived. Enhanced the digital image in the midst of vagueness and uncertainty of images through Type I fuzzy set and index of fuzziness is measured to reach the optimal clarification of decision making. These results are incorporated to enhance especially the medical image.

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